

3.3. TWINNING OF CRYSTALS

small lattice indices are listed, for which twinning has never been observed.

Rutile: Here nearly twenty 'permissible' $(hkl)/[uvw]$ combinations with $\omega \leq 6^\circ$, $[j] \leq 6$ occur. For the frequent (101) reflection twins, five permissible cases are calculated, of which two are given in the table. For the rare (301) reflection twins, only the one case listed, with high obliquity $\omega = 5.4^\circ$, is permissible. For the further two cases of low obliquity and lattice index [5], twins are not known. Among them is one case of (strict) 'reticular merohedry', (210) or (130), with $\omega = 0$ and $[j] = 5$ (cf. Fig. 3.3.8.1).

Quartz: The various quartz twins with inclined axes were studied extensively by Friedel (1923). The two most frequent cases, the Japanese (1122) twin (called La Gardette twin by Friedel) and the (1011) Esterel twin, are considered here. In both cases, several lattice pseudosymmetries occur. Following Friedel, those with the smallest lattice index, but relatively high obliquity close to 6° are listed in the table. Again, a twin of (strict) 'reticular merohedry' with $\omega = 0$ and $[j] = 7$ does not occur [cf. Section 3.3.9.2.3, Example (2)].

Staurolite: Both twin laws occurring in nature, (031) and (231), exhibit small obliquities but rather high lattice indices [6] and [12]. The frequent (231) 60° twin with $[j] = 12$ falls far outside the 'permissible' range. The further two planes listed in the table, (201) and (101), exhibit favourably small obliquities and lattice indices, but do not form twins. The existing (031) and (231) twins of staurolite are discussed again in Section 3.3.9.2 under the aspect of 'reticular pseudo-merohedry'.

Calcite: For calcite, 19 lattice pseudosymmetries obeying Friedel's 'permissible criteria' are calculated. Again, only a few are mentioned here (indices referred to the structural cell). For the primary deformation twin (0118), e-twin after Bueble & Schmahl (1999), cf. Section 3.3.10.2.2, Example (5), one permis-

sible lattice pseudosymmetry with small obliquity 0.59 but high lattice index [5] is found. For the less frequent secondary deformation twin (1014), r-twin, the situation is similar. The planes (0112) and (1011) permit small obliquities and lattice indices $\leq [5]$, but do not appear as twin planes.

The discussion of the examples in Table 3.3.8.2 shows that, with one exception [staurolite (231) twin], the obliquities and lattice indices of common twins fall within the $\omega/[j]$ limits accepted for lattice pseudosymmetry. Three aspects, however, have to be critically evaluated:

(i) For most of the lattice planes (hkl) , several pseudo-normal rows $[uvw]$ with different values of ω and $[j]$ within the $6^\circ/[6]$ limit occur, and *vice versa*. Friedel (1923) discussed this in his theory of quartz twinning. He considers the $(hkl)/[uvw]$ combination with the smallest lattice index as responsible for the observed twinning.

(ii) Among the examples given in the table, low-index $(hkl)/[uvw]$ combinations with more favourable $\omega/[j]$ values than for the existing twins can be found that never form twins. A prediction of twins on the basis of 'lattice control' alone, characterized by low ω and $[j]$ values, would fail in these cases.

(iii) All examples in the table were derived solely from lattice geometry, none from structural relations or other physical factors.

Note. As a mathematical alternative to the term 'obliquity', another more general measure of the deviation suffered by the twin lattice in crossing the twin boundary was presented by Santoro (1974, equation 36). This measure is the difference between the metric tensors of lattice 1 and of lattice 2, the latter after retransformation by the existing or assumed twin operation (or more general orientation operation).

Table 3.3.8.2. Examples of calculated obliquities ω and lattice indices $[j]$ for selected $(hkl)/[uvw]$ combinations and their relation to twinning

Calculations were performed with the program *OBLIQUE* written by Le Page (1999, 2002).

Crystal	(hkl)	Pseudo-normal $[uvw]$	Obliquity $[\omega]$	Lattice index $[j]$	Remark
Gypsum $A2/a$ $a = 6.51, b = 15.15, c = 6.28 \text{ \AA}$ $\beta = 127.5^\circ$	(100)	[302] [805]	2.47 0.42	3 4	Dovetail twin (very frequent)
	(001)	[203] [305]	5.92 0.95	3 5	Montmartre twin (less frequent)
	(101)	[101]	2.60	2	No twin
	(111)	[314]	1.35	4	No twin
Rutile $P4_2/mnm$ $a = 4.5933, c = 2.9592 \text{ \AA}$	(101)	[102] [307]	5.02 0.84	3 5	Frequent twin
	(301)	[101]	5.43	2	Rare twin
	(201)	[304]	2.85	5	No twin
	(210) or (130)	[210] or [130]	0	5	No twin
Quartz $P3_121$ $a = 4.9031, c = 5.3967 \text{ \AA}$	(1122)	[111]	5.49	2	Japanese twin (La Gardette) (rare)
	(1011)	[211]	5.76	3	Esterel twin (rare)
	(1012)	[212]	5.76	3	Sardinia twin (very rare)
	(2130) or (1450)	[540] or [230]	0	7	No twin
Staurolite $C2/m$ $a = 7.781, b = 16.620, c = 5.656 \text{ \AA}$ $\beta = 90.00^\circ$	(031)	[013]	1.19	6	90° twin (rare)
	(231)	[313]	0.90	12	60° twin (frequent)
	(201)	[101]	0.87	3	No twin
	(101)	[102]	0.87	3	No twin
Calcite $R3c$ $a = 4.989, c = 17.062 \text{ \AA}$ [hexagonal axes, structural X-ray cell; cf. Section 3.3.10.2.2, Example (5)]	(0112)	[5,10,1] [7,14,2] [481]	5.31 2.57 0.59	2 3 5	No twin
	(1014)	[421]	0.74	4	Rare deformation twin (r-twin)
	(0118)	[121]	0.59	5	Frequent deformation twin (e-twin)
	(1011)	[14.7.1]	1.54	5	No twin