

3. PHASE TRANSITIONS, TWINNING AND DOMAIN STRUCTURES

Table 3.4.4.7. Sectional layer groups and twin (wall) symmetries of the twin ($S_1|[100]; sd|S_3$) in a calomel crystal

Location sd	$\overline{\mathcal{F}}_{13}$		$\overline{\mathcal{J}}_{13}$		T_{13}		Classification
	Standard	Non-coordinate	Standard	Non-coordinate	Standard	Non-coordinate	
$\frac{1}{4}d, \frac{3}{4}d$	$p12/m1$	$p2_z/m_z$	$pmma$	$pm_z^*m_z a_x^*$	$p2_1ma$	$p2_{z1}^*m_z a_x^*$	SR
$0d, \frac{1}{2}d$	$p12/m1^\dagger$	$p2_z/m_z^\dagger$	$pmmm^\dagger$	$pm_z^*m_z m_x^\dagger$	$p2nm$	$p2_{zy}^*m_z m_x^*$	SR
sd	$p1m1$	pm_z	$pm2$	$p2_x^*m_y^*m_z$	$p1m1$	pm_z	AR*

† Shift of origin $b_i/2$.

preserving and side-reversing operations, whereas for any other location sd these layer groups are trivial (face) layer groups consisting of side-preserving operations only and are, therefore, also called *floating groups* in the direction d (IT E, 2002).

The wall (twin) symmetry T_{13} can be easily deduced from sectional layer groups $\overline{\mathcal{F}}_{13}$ and $\overline{\mathcal{J}}_{13}$: the floating group $\widehat{\mathcal{F}}_{13}$ is just the sectional layer group $\overline{\mathcal{F}}_{13}$ at a general location, $\widehat{\mathcal{F}}_{13} = \overline{\mathcal{F}}_{13}(sd) = pm_z$. Two other generators in the group symbol of T_{13} are non-trivial twinning operations (underlined with a star) of $\overline{\mathcal{J}}_{13}$. The classification in the last column of Table 3.4.4.7 is defined in Table 3.4.4.3.

Local symmetry exerts constraints on possible displacements of the atoms within a wall. The site symmetry of atoms in a wall of zero thickness, or at the central plane of a finite-thickness domain wall, are defined by the layer group T_{13} . The site symmetry of the off-centre atoms at $0 < |\xi| < \infty$ are determined by floating group $\widehat{\mathcal{F}}_{13}$ and the limiting structures at $\xi \rightarrow -\infty$ and $\xi \rightarrow \infty$ by space groups \mathcal{F}_1 and \mathcal{F}_3 , respectively. A reasonable condition that the displacements of atoms change continuously if one passes through the wall from $\xi \rightarrow -\infty$ to $\xi \rightarrow \infty$ allows one to deduce a qualitative picture of the displacements within a wall.

Symmetry groups of domain pairs, sectional layer groups and the twin symmetry have been derived in the parent clamping approximation (PCA) (see Section 3.4.2.5). As can be seen from Fig. 3.4.3.5, a relaxation process, accompanying a lifting of this approximation, consists of a simple shear (shear vector parallel to q) and an elongation (or contraction) in the domain wall along the shear direction (change of the vector AB_0 into the vector AB_1^\dagger). These deformations influence neither the layer group T_{13} nor its floating group $\widehat{\mathcal{F}}_{13}$. Hence the wall (twin) symmetry T_{13} derived in the parent clamping approximation expresses also the symmetry of a ferroelastic domain wall (twin) with nonzero spontaneous shear unless the simple shear is accompanied by a reshuffling of atoms or molecules in both domains. This useful statement holds for any ferroelastic domain wall (twin).

A microscopic structure of the ferroelastic domain wall in two symmetrically prominent positions is depicted in Fig. 3.4.4.11. For better recognition, displacements of molecules are exaggerated

and the changes of the displacement lengths are neglected. Since the symmetry of all groups involved contains a reflection m_z , the atomic shifts are confined to planes (001). It can be seen in the figure that when one moves through the wall in the direction $[110]$ or $[1\bar{1}0]$, the vector of the molecular shift experiences rotations through $\frac{1}{2}\pi$ about the c' direction in opposite senses for the ‘black’ and ‘white’ molecules.

The ‘black’ molecules in the central layer at location $\frac{1}{4}d$ or $\frac{3}{4}d$ [wall (a) on the left-hand side of Fig. 3.4.4.11] exhibit nearly antiparallel displacements perpendicular to the wall. Strictly perpendicular shifts would represent ‘averaged’ displacements compatible with the layer symmetry $\overline{\mathcal{J}}_{13} = p2_{z1}^*m_z a_x^*$, which is, however, broken by a simple shear that decreases the symmetry to $T_{13} = p2_{z1}^*m_z a_x^*$, which does not require perpendicular displacements of ‘black’ molecules.

The wall with central plane location $0d$ or $\frac{1}{2}d$ (Fig. 3.4.4.11b) has symmetry $T_{13} = p2_{zy}^*m_z m_x^*$, which restricts displacements of ‘white’ molecules of the central layer to the y direction only; the ‘averaged’ displacements compatible with $\overline{\mathcal{J}}_{13} = pm_y^*m_z m_x^*$ (origin shift $b'/2$) would have equal lengths of shifts in the $+y$ and $-y$ directions, but the relaxed central layer with symmetry $T_{13} = p2_{zy}^*m_z m_x^*$ allows unequal shifts in the $-y$ and $+y$ directions.

Walls (a) and (b) with two different prominent locations have different layer symmetries and different structures of the central layer. These two walls have extremal energy, but symmetry cannot decide which one has the minimum energy. The two walls have the same polar point-group symmetry $m_z^*2_{zy}^*m_z$, which permits a spontaneous polarization along y .

Similar analysis of the displacement and ordering fields in domain walls has been performed for KSCN crystals (Janovec *et al.*, 1989), sodium superoxide NaO_2 (Zieliński, 1990) and for the simple cubic phase of fullerene C_{60} (Saint-Grégoire *et al.*, 1997).

3.4.5. Glossary

Note: the correspondence between contracted Greek indices and the Cartesian vector components used in Sections 3.1.3, in the present chapter and in the software *GI★KoBo-1*, is defined in the following way:

Cartesian components	11	22	33	23, 32	31, 13	12, 21
Contracted notation	1	2	3	4	5	6

In this designation, coefficients with contracted indices 4, 5, 6 appear two times, e.g. index 4 replaces yz in one coefficient and zy in the other coefficient. With this convention, the coefficients transform in tensor space as vector components, but some coefficients differ from the usual matrix notation (Voigt matrices) by numerical factors [see Section 1.1.4.10; Nye (1985); Sirotnin & Shaskolskaya, Appendix E (1982)].

- (a) Objects
- B_m domain region
- d scanning vector (basis vector of a scanning group)
- $D_i(S_k, B_m)$ the i th domain, with domain state S_k in the m th domain region B_m
- GS_1 G -orbit of principal single-domain states

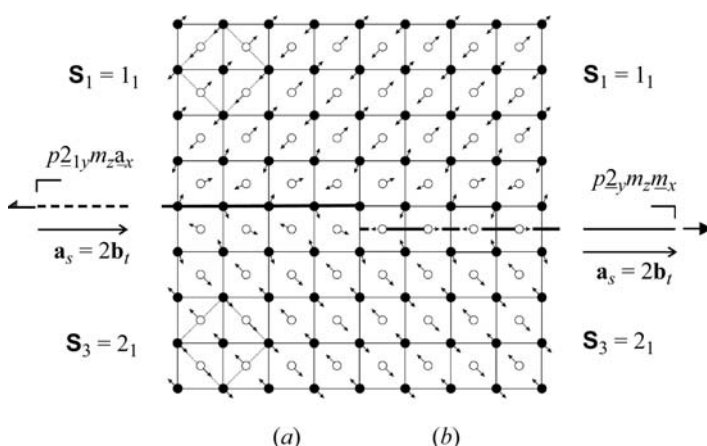


Fig. 3.4.4.11. Microscopic structure of a ferroelastic domain wall in calomel. (a) and (b) show a domain wall at two different locations with two different layer groups and two different structures of the central planes.

3.4. DOMAIN STRUCTURES

$G(\mathbf{S}_1, \mathbf{S}_j)$	G -orbit of domain pairs	\bar{J}_{ik}	sectional layer group of J_{ik}
$G(\mathbf{S}_1 \mathbf{n} \mathbf{S}_j)$	G -orbit of simple domain twins	\hat{J}_{ik}	face group, trivial subgroup, floating subgroup of sectional group of J_{ik}
\mathbf{n}	normal to a plane p	r_{ik}^*	symmetry operation of \bar{J}_{ik} that exchanges \mathbf{S}_i and \mathbf{S}_k
p	plane of a domain wall, domain wall plane	\underline{s}_{ik}	symmetry operation of \bar{J}_{ik} that inverts \mathbf{n} into $-\mathbf{n}$
$\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_i, \dots$	secondary ferroic single-domain states	\underline{t}_{ik}^*	symmetry operation of \bar{J}_{ik} that exchanges \mathbf{S}_i and \mathbf{S}_k and inverts \mathbf{n} into $-\mathbf{n}$
$\mathbf{R}_1^+, \mathbf{R}_1^-, \mathbf{R}_2^+, \mathbf{R}_2^-, \dots$	disoriented secondary ferroic domain states	$\mathbf{T}_{ik}(\mathbf{n})$	symmetry group of the twin $\mathbf{T}_{ik}(\mathbf{n})$
$\mathbf{sd} (0 \leq s < 1)$	location of a plane in crystal lattice	$\mathbf{W}_{ik}(\mathbf{n})$	symmetry group of the domain wall $\mathbf{W}_{ik}(\mathbf{n})$
$\mathbf{S}_1, \mathbf{S}_2, \dots, \mathbf{S}_i, \dots$	principal single-domain states (orientation states, variants)	\mathcal{T}_i	translational subgroup of \mathcal{F}_i
$\mathbf{S}_1^+, \mathbf{S}_1^-, \mathbf{S}_2^+, \mathbf{S}_2^-, \dots$	disoriented domain states	\mathcal{T}_{ik}	translational subgroup of \mathcal{F}_{ik}
$\mathbf{S}_1^-, \mathbf{S}_2^-, \dots, \mathbf{S}_i^-, \dots$	basic (microscopic) single-domain states (structural variants)		
$(\mathbf{S}_i, \mathbf{S}_k)$	ordered domain pair = ordered pair of domain states \mathbf{S}_i and \mathbf{S}_k		
$\{\mathbf{S}_i, \mathbf{S}_k\}$	unordered domain pair = unordered pair of domain states \mathbf{S}_i and \mathbf{S}_k		
$(\mathbf{S}_i \mathbf{n} \mathbf{S}_k)$	simple domain twin formed from single-domain states		
$(\mathbf{S}_i^+ \mathbf{n} \mathbf{S}_k^-)$	simple ferroelastic domain twin with a compatible domain wall		
$[\mathbf{S}_i \mathbf{n} \mathbf{S}_k]$	domain wall in the simple twin $(\mathbf{S}_i \mathbf{n} \mathbf{S}_k)$		
$\mathbf{T}_{ik}(\mathbf{n})$ or \mathbf{T}_{ik}	simple domain twin – short symbol		
$\mathbf{W}_{ik}(\mathbf{n})$ or \mathbf{W}_{ik}	domain wall – short symbol		
φ	shear angle, obliquity		
$\pm \frac{1}{2}\varphi$	disorientation angle of a domain state		

(b) Symmetry groups – point groups in a continuum description and space groups in a microscopic description

F	point-group symmetry of the ferroic phase (domain state not specified)
\mathcal{F}	space-group symmetry of the ferroic phase (domain state not specified)
F_i	point-group symmetry of a principal domain state \mathbf{S}_i
\mathcal{F}_i	space-group symmetry of a basic (microscopic) domain state \mathbf{S}_i
F_{ik}	point-group symmetry (stabilizer in G) of the ordered domain pair $(\mathbf{S}_i, \mathbf{S}_k)$
\mathcal{F}_{ik}	space-group symmetry (stabilizer in \mathcal{G}) of the ordered domain pair $(\mathbf{S}_i, \mathbf{S}_k)$
\bar{F}_{ik}	sectional layer group of F_{ik}
\hat{F}_{ik}	face group, trivial layer group, scanning group of F_{ik}
Fam G	crystal family of the group G
G	point-group symmetry of the parent phase
\mathcal{G}	space-group symmetry of the parent phase
g	point-group symmetry operation of the group $G(\mathcal{G})$
\mathfrak{g}	space-group symmetry operation of the group \mathcal{G}
g_{ik}	switching operation in domain pair $(\mathbf{S}_i, \mathbf{S}_k)$, transforms \mathbf{S}_i into \mathbf{S}_k
g_{ik}^*	transposing operation in domain pair $(\mathbf{S}_i, \mathbf{S}_k)$, exchanges \mathbf{S}_i and \mathbf{S}_k , twinning operation of a non-ferroelastic domain pair $(\mathbf{S}_i, \mathbf{S}_k)$
$I_G(\mathbf{S}_i)$	stabilizer (isotropy group) of \mathbf{S}_i in G
$\mathcal{I}_{\mathcal{G}}(\mathbf{S}_i)$	stabilizer (isotropy group) of \mathbf{S}_i in \mathcal{G}
J_{ik}	point-group symmetry (stabilizer in G) of the unordered domain pair $\{\mathbf{S}_i, \mathbf{S}_k\}$
J_{ik}^*	point-group symmetry (stabilizer in G) of a completely transposable domain pair $\{\mathbf{S}_i, \mathbf{S}_k\}$
\mathcal{J}_{ik}	space-group symmetry (stabilizer in \mathcal{G}) of the unordered domain pair $\{\mathbf{S}_i, \mathbf{S}_k\}$
K_{ik}	twinning group of the domain pair $(\mathbf{S}_i, \mathbf{S}_k)$
K_{ik}^*	twinning group of a completely transposable domain pair $(\mathbf{S}_i, \mathbf{S}_k)$
L_i	intermediate group, $F_i \in L_i \in G$

(c) Components of property tensors

ε	enantiomorphism
P_i	polarization
u_μ	strain
g_μ	optical activity
$d_{i\mu}$	piezoelectricity
A_{iv}	electrogyration
$s_{\mu\nu}$	linear elasticity
$Q_{\mu\nu}$	electrostriction

$i = 1, 2, 3; \mu, \nu = 1, 2, \dots, 6.$

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