Table 1.10.5.3. *The representation matrices for* Γ_3

The representation matrices for Γ_3 are the same as for Γ_2. Correspondences are given as pairs i, j: $\Gamma_3(R_i) = \Gamma_2(R_j)$.

i	j	i	j	i	j	i	j	i	j	i	j
1	1	11	21	21	5	31	42	41	29	51	48
2	14	12	16	22	6	32	45	42	39	52	54
3	23	13	17	23	8	33	36	43	33	53	46
4	15	14	4	24	10	34	27	44	30	54	50
5	25	15	2	25	11	35	26	45	38	55	52
6	24	16	13	26	34	36	28	46	49	56	57
7	19	17	12	27	35	37	31	47	53	57	59
8	20	18	7	28	43	38	40	48	51	58	56
9	18	19	9	29	44	39	37	49	47	59	58
10	22	20	3	30	41	40	32	50	55	60	60

Table 1.10.5.4. *Number of free parameters for some tensors and their symmetry groups*

Tensor	Elements	222(−1−11)	622(1−1−1)	$\bar{5}\bar{3}m(\bar{5}^2\bar{3}m)$
Metric	g_{ij}	5	4	2
Elasticity	e_{ijkl}	17	10	5
Phonon elasticity	e^{EE}_{ijkl}	9	5	2
Phason elasticity	e^{II}_{ijkl}	3	2	2
Phonon–phason elasticity	e^{EI}_{ijkl}	5	3	1

I. The direct products with $\mathbb{Z}_2$ then follow easily. Although these direct products of a group K with $\mathbb{Z}_2$ do not belong to the isomorphism class of K, their irreducible representations are nevertheless given in the table for K because these irreducible representations have the same labels as those for K apart from an additional subindex u. The reresentations of the subgroup K of $K \times \mathbb{Z}_2$ are the same as for K itself, those for the cosets get an additional minus sign. In the tables, the characters for the groups $K \times \mathbb{Z}_2$ are separated from those for K by a horizontal rule. In addition to the characters are given the realizations of crystallographic point groups, and the irreducible components of the vector representations in direct space V_E and internal space V_I for these realizations. The vector representation in V_I is called the perpendicular representation.

In Table 1.10.5.2 the representation matrices for the irreducible representations in more than one dimension are given (one-dimensional representations are just the characters). For the cyclic groups there are only one-dimensional representations, for the dihedral groups there are one- and two-dimensional irreducible representations. There are four irreducible representations of I of dimension larger than one. The four- and five-dimensional ones are given as integer representations. They form crystallographic groups in 4D and 5D. The two three-dimensional representations have the same matrices. The elements, however, are connected by an outer automorphism. That means that the ith element R_i is represented by $\Gamma_2(R_i)$ in the representation Γ_2, and by $\Gamma_3(R_i) = \Gamma_2(\varphi R_i)$ in Γ_3. The element φR_i is another element R_j. The corresponding j for each i is given in Table 1.10.5.3. Examples of physical tensors are given in Table 1.10.5.4 with their respective numbers of free parameters as determined using representations of three nD point groups.

References

Bak, P. (1985). *Symmetry, stability, and elastic properties of icosahedral incommensurate crystals. Phys. Rev. B*, **32**, 5764–5772.

Beest, B. W. van, Janner, A. & Blinc, R. (1983). *^{87}Rb electric field gradient tensors and the symmetry of the incommensurate phase in Rb_2ZnBr_4 and Rb_2ZnCl_4. J. Phys. C*, **16**, 5409–5416.

Ding, D.-H., Yang, W.-G., Hu, C.-Z. & Wang, R.-H. (1993). *Generalized elasticity theory of quasicrystals. Phys. Rev. B*, **48**, 7003–7010.

International Tables for Crystallography (2004). Vol. C, *Mathematical, Physical and Chemical Tables*, edited by E. Prince. Dordrecht: Kluwer Academic Publishers.

Janner, A. & Janssen, T. (1980). *Symmetry of incommensurate crystal phases. I. Commensurate basic structures*. *Acta Cryst.* A**36**, 399–408.

Janssen, T. (1988). *Aperiodic crystals: a contradictio in terminis? Phys. Rep.* **168**, 57–113.

Janssen, T. (1997). *Tensor properties of incommensurate phases*. *Ferroelectrics*, **203**, 309–322.

Janssen, T., Birman, J. L., Dénoyer, F., Koptsik, V. A., Verger-Gaugry, J. L., Weigel, D., Yamamoto, A., Abrahams, S. C. & Kopsky, V. (2002). *Report of a subcommittee on the nomenclature of n-dimensional crystallography. II. Symbols for arithmetic crystal classes, Bravais classes and space groups. Acta Cryst.* A**58**, 605–621.

Janssen, T., Birman, J. L., Koptsik, V. A., Senechal, M., Weigel, D., Yamamoto, A., Abrahams, S. C. & Hahn, Th. (1999). *Report of a subcommittee on the nomenclature of n-dimensional crystallography. I. Symbols for point-group transformations, families, systems and geometric crystal classes. Acta Cryst*. A**55**, 761–782.

Janssen, T., Chapuis, G. & de Boissieu, M. (2007). *Aperiodic Crystals, from Incommensurate Phases to Quasicrystals*. Oxford University Press.

Janssen, T. & Janner, A. (1987). *Incommensurability in crystals. Adv. Phys.* **36**, 519–624.

Lubensky, T. C., Ramaswamy, S. & Toner, J. (1985). *Hydrodynamics of icosahedral quasicrystals. Phys. Rev. B*, **32**, 7444–7452.

Perez-Mato, J. M., Ribeiro, J. L., Petricek, V. & Aroyo, M. I. (2012). *Magnetic superspace groups and symmetry constraints in incommensurate magnetic phases*. *J. Phys. Condens. Matter*, **24**, 163201.

Schobinger-Papamantellos, P., Janssen, T. & Buschow, K. H. J. (1993). *Thermal variation of magnetic phases in TbSi as observed by neutron diffraction*. *J. Magn. Magn. Mater.* **127**, 115–128.

Shechtman, D., Blech, I., Gratias, D. & Cahn, J. W. (1984). *Metallic phase with long-range orientational order and no translational symmetry. Phys. Rev. Lett.* **53**, 1951–1953.

Socolar, J. E. S., Lubensky, T. C. & Steinhardt, P. J. (1986). *Phonons, phasons and dislocations in quasicrystals. Phys. Rev. B*, **34**, 3345–3360.