International Tables for Crystallography (2013). Vol. D, Section 1.10.5, pp. 262–268.

Table 1.10.5.2. *Matrices of the irreducible representations of dimension* $d \geq 2$ *corresponding to the irreps of Table 1.10.5.1*

(*a*) D_5

Representation	$D(\alpha^p)$	$D(\beta)$
Γ_3	$\begin{pmatrix} \cos(2\pi p/5) & -\sin(2\pi p/5) \\ \sin(2\pi p/5) & \cos(2\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_4	$\begin{pmatrix} \cos(4\pi p/5) & -\sin(4\pi p/5) \\ \sin(4\pi p/5) & \cos(4\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(*b*) D_8

Representation	$D(\alpha^p)$	$D(\beta)$
Γ_5	$\begin{pmatrix} \cos(\pi p/4) & -\sin(\pi p/4) \\ \sin(\pi p/4) & \cos(\pi p/4) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_6	$\begin{pmatrix} \cos(\pi p/2) & -\sin(\pi p/2) \\ \sin(\pi p/2) & \cos(\pi p/2) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_7	$\begin{pmatrix} \cos(3\pi p/4) & -\sin(3\pi p/4) \\ \sin(3\pi p/4) & \cos(3\pi p/4) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(*c*) D_{10}

Representation	$D(\alpha^p)$	$D(\beta)$
Γ_5	$\begin{pmatrix} \cos(\pi p/5) & -\sin(\pi p/5) \\ \sin(\pi p/5) & \cos(\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_6	$\begin{pmatrix} \cos(2\pi p/5) & -\sin(2\pi p/5) \\ \sin(2\pi p/5) & \cos(2\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_7	$\begin{pmatrix} \cos(3\pi p/5) & -\sin(3\pi p/5) \\ \sin(3\pi p/5) & \cos(3\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_8	$\begin{pmatrix} \cos(4\pi p/5) & -\sin(4\pi p/5) \\ \sin(4\pi p/5) & \cos(4\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(*d*) D_{12}

Representation	$D(\alpha^p)$	$D(\beta)$
Γ_5	$\begin{pmatrix} \cos(\pi p/6) & -\sin(\pi p/6) \\ \sin(\pi p/6) & \cos(\pi p/6) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_6	$\begin{pmatrix} \cos(\pi p/3) & -\sin(\pi p/3) \\ \sin(\pi p/3) & \cos(\pi p/3) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_7	$\begin{pmatrix} \cos(\pi p/2) & -\sin(\pi p/2) \\ \sin(\pi p/2) & \cos(\pi p/2) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_8	$\begin{pmatrix} \cos(2\pi p/3) & -\sin(2\pi p/3) \\ \sin(2\pi p/3) & \cos(2\pi p/3) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_9	$\begin{pmatrix} \cos(5\pi p/6) & -\sin(5\pi p/6) \\ \sin(5\pi p/6) & \cos(5\pi p/6) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

In linear approximation the coupling between distortion and magnetic moments may be given by the energy expression

$$U = \sum_{ijk} C_{ijk}\sigma_{ij}M_k. \qquad (1.10.4.31)$$

The tensor C_{ijk} transforms with the product of the representation corresponding to the strain tensor σ and the pseudovector representation corresponding to M. For the former one may distinguish a phonon and a phason component, respectively e and f. The first (e) transforms with the symmetrized square of the vector representation v_E and the second (f) with the product $v_E \oplus v_I$ of the vector representations in physical and internal spaces. If this tensor representation contains the identity representation, then there are one or more independent components and the coupling is possible. If the identity representation is not present, the coupling is forbidden by symmetry.

1.10.5. Tables

In this section are presented the irreducible representations of point groups of quasiperiodic structures up to rank six that do not occur as three-dimensional crystallographic point groups.

Table 1.10.5.1 gives the characters of the point groups C_n with n = 5, 8, 10, 12, D_n with n = 5, 8, 10, 12, and the icosahedral group

Table 1.10.5.2 (*cont.*)

(*e*) *I*. First column: numbering of the elements. $f = (1+\sqrt{5})/2$, $t = (\sqrt{5}-1)/2$. Horizontal rules separate conjugation classes.

No.	Order	Γ_2	Γ_4	Γ_5
1	1	$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$	$\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}$
2	5	$\begin{pmatrix}1/2&t/2&-f/2\\t/2&f/2&1/2\\f/2&-1/2&t/2\end{pmatrix}$	$\begin{pmatrix}0&0&0&-1\\1&0&0&-1\\0&1&0&-1\\0&0&1&-1\end{pmatrix}$	$\begin{pmatrix}1&0&0&0&-1\\0&0&0&0&-1\\0&1&0&0&-1\\0&0&1&0&-1\\0&0&0&1&-1\end{pmatrix}$
3	5	$\begin{pmatrix}1/2&-t/2&f/2\\-t/2&f/2&1/2\\f/2&-1/2&t/2\end{pmatrix}$	$\begin{pmatrix}0&0&1&-1\\1&0&0&-1\\0&0&0&-1\\0&1&0&-1\end{pmatrix}$	$\begin{pmatrix}0&-1&1&0&0\\0&-1&0&0&1\\0&-1&0&0&0\\0&-1&0&1&0\\1&-1&0&0&0\end{pmatrix}$
4	5	$\begin{pmatrix}1/2&t/2&f/2\\t/2&f/2&-1/2\\-f/2&1/2&t/2\end{pmatrix}$	$\begin{pmatrix}-1&1&0&0\\-1&0&1&0\\-1&0&0&1\\-1&0&0&0\end{pmatrix}$	$\begin{pmatrix}1&-1&0&0&0\\0&-1&1&0&0\\0&-1&0&1&0\\0&-1&0&0&1\\0&-1&0&0&0\end{pmatrix}$
5	5	$\begin{pmatrix}t/2&-f/2&1/2\\f/2&1/2&t/2\\-1/2&t/2&f/2\end{pmatrix}$	$\begin{pmatrix}0&-1&0&0\\0&-1&0&1\\1&-1&0&0\\0&-1&1&0\end{pmatrix}$	$\begin{pmatrix}0&1&0&-1&0\\0&0&0&-1&1\\0&0&1&-1&0\\1&0&0&-1&0\\0&0&0&-1&0\end{pmatrix}$
6	5	$\begin{pmatrix}f/2&-1/2&-t/2\\1/2&t/2&f/2\\-t/2&-f/2&1/2\end{pmatrix}$	$\begin{pmatrix}0&1&-1&0\\0&0&-1&0\\0&0&-1&1\\1&0&-1&0\end{pmatrix}$	$\begin{pmatrix}0&1&0&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&0&1&0&0\\1&0&0&0&0\end{pmatrix}$
7	5	$\begin{pmatrix}f/2&1/2&t/2\\-1/2&t/2&f/2\\t/2&-f/2&1/2\end{pmatrix}$	$\begin{pmatrix}0&-1&0&1\\0&-1&1&0\\1&-1&0&0\\0&-1&0&0\end{pmatrix}$	$\begin{pmatrix}-1&0&1&0&0\\-1&1&0&0&0\\-1&0&0&0&1\\-1&0&0&0&0\\-1&0&0&1&0\end{pmatrix}$
8	5	$\begin{pmatrix}t/2&f/2&-1/2\\-f/2&1/2&t/2\\1/2&t/2&f/2\end{pmatrix}$	$\begin{pmatrix}-1&0&1&0\\-1&0&0&0\\-1&0&0&1\\-1&1&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&1&-1\\1&0&0&0&-1\\0&0&1&0&-1\\0&0&0&0&-1\\0&1&0&0&-1\end{pmatrix}$
9	5	$\begin{pmatrix}t/2&f/2&1/2\\-f/2&1/2&-t/2\\-1/2&-t/2&f/2\end{pmatrix}$	$\begin{pmatrix}0&0&-1&0\\0&0&-1&1\\0&1&-1&0\\1&0&-1&0\end{pmatrix}$	$\begin{pmatrix}-1&0&0&1&0\\-1&0&1&0&0\\-1&0&0&0&0\\-1&1&0&0&0\\-1&0&0&0&1\end{pmatrix}$
10	5	$\begin{pmatrix}f/2&1/2&-t/2\\-1/2&t/2&-f/2\\-t/2&f/2&1/2\end{pmatrix}$	$\begin{pmatrix}0&-1&0&1\\1&-1&0&0\\0&-1&0&0\\0&-1&1&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&1\\1&0&0&0&0\\0&0&0&1&0\\0&1&0&0&0\\0&0&1&0&0\end{pmatrix}$
11	5	$\begin{pmatrix}1/2&-t/2&-f/2\\-t/2&f/2&-1/2\\f/2&1/2&t/2\end{pmatrix}$	$\begin{pmatrix}0&1&-1&0\\0&0&-1&1\\1&0&-1&0\\0&0&-1&0\end{pmatrix}$	$\begin{pmatrix}0&0&-1&0&1\\0&0&-1&0&0\\1&0&-1&0&0\\0&0&-1&1&0\\0&1&-1&0&0\end{pmatrix}$
12	5	$\begin{pmatrix}f/2&-1/2&t/1\\1/2&t/2&-f/2\\t/2&f/2&1/2\end{pmatrix}$	$\begin{pmatrix}0&0&1&-1\\0&0&0&-1\\0&1&0&-1\\1&0&0&-1\end{pmatrix}$	$\begin{pmatrix}0&0&0&-1&0\\0&1&0&-1&0\\1&0&0&-1&0\\0&0&0&-1&1\\0&0&1&-1&0\end{pmatrix}$

Table 1.10.5.2 (*cont.*)

No.	Order	Γ_2	Γ_4	Γ_5
13	5	$\begin{pmatrix} t/2 & -f/2 & -1/2 \\ f/2 & 1/2 & -t/2 \\ 1/2 & -t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$
14	5	$\begin{pmatrix} -t/2 & f/2 & -1/2 \\ f/2 & 1/2 & t/2 \\ 1/2 & -t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix}$
15	5	$\begin{pmatrix} -t/2 & f/2 & 1/2 \\ f/2 & 1/2 & -t/2 \\ -1/2 & t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix}$
16	5	$\begin{pmatrix} -f/2 & 1/2 & -t/2 \\ -1/2 & -t/2 & f/2 \\ t/2 & f/2 & 1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \end{pmatrix}$
17	5	$\begin{pmatrix} -t/2 & -f/2 & 1/2 \\ -f/2 & 1/2 & t/2 \\ -1/2 & -t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}$
18	5	$\begin{pmatrix} -t/2 & -f/2 & -1/2 \\ -f/2 & 1/2 & -t/2 \\ 1/2 & t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}$
19	5	$\begin{pmatrix} -f/2 & -1/2 & t/2 \\ 1/2 & -t/2 & f/2 \\ -t/2 & f/2 & 1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \end{pmatrix}$
20	5	$\begin{pmatrix} -f/2 & 1/2 & t/2 \\ -1/2 & -t/2 & -f/2 \\ -t/2 & -f/2 & 1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$
21	5	$\begin{pmatrix} 1/2 & -t/2 & f/2 \\ t/2 & -f/2 & -1/2 \\ f/2 & 1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \end{pmatrix}$
22	5	$\begin{pmatrix} 1/2 & -t/2 & -f/2 \\ t/2 & -f/2 & 1/2 \\ -f/2 & -1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$
23	5	$\begin{pmatrix} 1/2 & t/2 & f/2 \\ -t/2 & -f/2 & 1/2 \\ f/2 & -1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix}$
24	5	$\begin{pmatrix} 1/2 & t/2 & -f/2 \\ -t/2 & -f/2 & -1/2 \\ -f/2 & 1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

Table 1.10.5.2 (*cont.*)

No.	Order	Γ_2	Γ_4	Γ_5
25	5	$\begin{pmatrix} -f/2 & -1/2 & -t/2 \\ 1/2 & -t/2 & -f/2 \\ t/2 & -f/2 & 1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}$
26	3	$\begin{pmatrix} -1/2 & t/2 & -f/2 \\ -t/2 & f/2 & 1/2 \\ f/2 & 1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix}$
27	3	$\begin{pmatrix} -1/2 & -t/2 & f/2 \\ t/2 & f/2 & 1/2 \\ -f/2 & 1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \end{pmatrix}$
28	3	$\begin{pmatrix} -1/2 & t/2 & f/1 \\ -t/2 & f/2 & -1/2 \\ -f/2 & -1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \end{pmatrix}$
29	3	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$
30	3	$\begin{pmatrix} -1/2 & -t/2 & -f/2 \\ t/2 & f/2 & -1/2 \\ f/2 & -1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{pmatrix}$
31	3	$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \end{pmatrix}$
32	3	$\begin{pmatrix} f/2 & 1/2 & -t/2 \\ 1/2 & -t/2 & f/2 \\ t/2 & -f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \end{pmatrix}$
33	3	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix}$
34	3	$\begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{pmatrix}$
35	3	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix}$
36	3	$\begin{pmatrix} f/2 & -1/2 & t/2 \\ -1/2 & -t/2 & f/2 \\ -t/2 & -f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{pmatrix}$

Table 1.10.5.2 (*cont.*)

No.	Order	Γ_2	Γ_4	Γ_5
37	3	$\begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix}$
38	3	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}$
39	3	$\begin{pmatrix} f/2 & 1/2 & t/2 \\ 1/2 & -t/2 & -f/2 \\ -t/2 & f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{pmatrix}$
40	3	$\begin{pmatrix} -t/2 & -f/2 & 1/2 \\ f/2 & -1/2 & -t/2 \\ 1/2 & t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \end{pmatrix}$
41	3	$\begin{pmatrix} -t/2 & -f/2 & -1/2 \\ f/2 & -1/2 & t/2 \\ -1/2 & -t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix}$
42	3	$\begin{pmatrix} -t/2 & f/2 & 1/2 \\ -f/2 & -1/2 & t/2 \\ 1/2 & -t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix}$
43	3	$\begin{pmatrix} -t/2 & f/2 & -1/2 \\ -f/2 & -1/2 & -t/2 \\ -1/2 & t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}$
44	3	$\begin{pmatrix} f/2 & -1/2 & -t/2 \\ -1/2 & -t/2 & -f/2 \\ t/2 & f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \end{pmatrix}$
45	3	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \end{pmatrix}$
46	2	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$
47	2	$\begin{pmatrix} -f/2 & 1/2 & t/2 \\ 1/2 & t/2 & f/2 \\ t/2 & f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
48	2	$\begin{pmatrix} -f/2 & -1/2 & -t/2 \\ -1/2 & t/2 & f/2 \\ -t/2 & f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$

Table 1.10.5.2 (*cont.*)

No.	Order	Γ_2	Γ_4	Γ_5
49	2	$\begin{pmatrix} -f/2 & -1/2 & t/2 \\ -1/2 & t/2 & -f/2 \\ t/2 & -f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$
50	2	$\begin{pmatrix} -f/2 & 1/2 & -t/2 \\ 1/2 & t/2 & -f/2 \\ -t/2 & -f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \end{pmatrix}$
51	2	$\begin{pmatrix} t/2 & f/2 & -1/2 \\ f/2 & -1/2 & -t/2 \\ -1/2 & -t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{pmatrix}$
52	2	$\begin{pmatrix} t/2 & f/2 & 1/2 \\ f/2 & -1/2 & t/2 \\ 1/2 & t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$
53	2	$\begin{pmatrix} -1/2 & t/2 & f/2 \\ t/2 & -f/2 & 1/2 \\ f/2 & 1/2 & t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$
54	2	$\begin{pmatrix} -1/2 & t/2 & -f/2 \\ t/2 & -f/2 & -1/2 \\ -f/2 & -1/2 & t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$
55	2	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}$
56	2	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{pmatrix}$
57	2	$\begin{pmatrix} -1/2 & -t/2 & -f/2 \\ -t/2 & -f/2 & 1/2 \\ -f/2 & 1/2 & t/2 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix}$
58	2	$\begin{pmatrix} t/2 & -f/2 & -1/2 \\ -f/2 & -1/2 & t/2 \\ -1/2 & t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$
59	2	$\begin{pmatrix} t/2 & -f/2 & 1/2 \\ -f/2 & -1/2 & -t/2 \\ 1/2 & -t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$
60	2	$\begin{pmatrix} -1/2 & -t/2 & f/2 \\ -t/2 & -f/2 & -1/2 \\ f/2 & -1/2 & t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

Table 1.10.5.3. *The representation matrices for* Γ_3

The representation matrices for Γ_3 are the same as for Γ_2. Correspondences are given as pairs i, j: $\Gamma_3(R_i) = \Gamma_2(R_j)$.

i	j	i	j	i	j	i	j	i	j	i	j
1	1	11	21	21	5	31	42	41	29	51	48
2	14	12	16	22	6	32	45	42	39	52	54
3	23	13	17	23	8	33	36	43	33	53	46
4	15	14	4	24	10	34	27	44	30	54	50
5	25	15	2	25	11	35	26	45	38	55	52
6	24	16	13	26	34	36	28	46	49	56	57
7	19	17	12	27	35	37	31	47	53	57	59
8	20	18	7	28	43	38	40	48	51	58	56
9	18	19	9	29	44	39	37	49	47	59	58
10	22	20	3	30	41	40	32	50	55	60	60

Table 1.10.5.4. *Number of free parameters for some tensors and their symmetry groups*

Tensor	Elements	222(−1−11)	622(1−1−1)	$\bar{5}\bar{3}m(\bar{5}^2\bar{3}m)$
Metric	g_{ij}	5	4	2
Elasticity	e_{ijkl}	17	10	5
Phonon elasticity	e^{EE}_{ijkl}	9	5	2
Phason elasticity	e^{II}_{ijkl}	3	2	2
Phonon–phason elasticity	e^{EI}_{ijkl}	5	3	1

I. The direct products with $\mathbb{Z}_2$ then follow easily. Although these direct products of a group K with $\mathbb{Z}_2$ do not belong to the isomorphism class of K, their irreducible representations are nevertheless given in the table for K because these irreducible representations have the same labels as those for K apart from an additional subindex u. The reresentations of the subgroup K of $K \times \mathbb{Z}_2$ are the same as for K itself, those for the cosets get an additional minus sign. In the tables, the characters for the groups $K \times \mathbb{Z}_2$ are separated from those for K by a horizontal rule. In addition to the characters are given the realizations of crystallographic point groups, and the irreducible components of the vector representations in direct space V_E and internal space V_I for these realizations. The vector representation in V_I is called the perpendicular representation.

In Table 1.10.5.2 the representation matrices for the irreducible representations in more than one dimension are given (one-dimensional representations are just the characters). For the cyclic groups there are only one-dimensional representations, for the dihedral groups there are one- and two-dimensional irreducible representations. There are four irreducible representations of I of dimension larger than one. The four- and five-dimensional ones are given as integer representations. They form crystallographic groups in 4D and 5D. The two three-dimensional representations have the same matrices. The elements, however, are connected by an outer automorphism. That means that the ith element R_i is represented by $\Gamma_2(R_i)$ in the representation Γ_2, and by $\Gamma_3(R_i) = \Gamma_2(\varphi R_i)$ in Γ_3. The element φR_i is another element R_j. The corresponding j for each i is given in Table 1.10.5.3. Examples of physical tensors are given in Table 1.10.5.4 with their respective numbers of free parameters as determined using representations of three nD point groups.

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