

1.1. INTRODUCTION TO THE PROPERTIES OF TENSORS

where a new piezoelectric tensor is introduced, e_{ijk} . Its components can be represented as a 3×9 matrix:

$$\begin{pmatrix} \mathbf{1} \\ - \\ \mathbf{2} \\ - \\ \mathbf{3} \end{pmatrix}.$$

Both sides of relation (1.1.4.8) remain unchanged if the indices i and j are interchanged, on account of the symmetry of the stress tensor. This shows that

$$e_{ijk} = e_{jik}.$$

Submatrices **2** and **3** are equal. One introduces here a two-index notation through the relation $e_{\alpha k} = e_{ijk}$, and the $e_{\alpha k}$ matrix can be written

$$\begin{pmatrix} \mathbf{1} \\ \mathbf{2+3} \end{pmatrix}.$$

The relation between the full and the reduced matrix is therefore different for the d_{ijk} and the e_{kij} tensors. This is due to the particular property of the strain Voigt matrix (1.1.4.6), and as a consequence the relations between nonzero components of the reduced matrices are different for certain point groups ($3, 32, 3m, \bar{6}, 62m$).

1.1.4.10.4. Independent components of the matrix associated with a third-rank polar tensor according to the following point groups

1.1.4.10.4.1. Triclinic system

- (i) Group 1: all the components are independent. There are 18 components.
- (ii) Group $\bar{1}$: all the components are equal to zero.

1.1.4.10.4.2. Monoclinic system

- (i) Group 2: twofold axis parallel to Ox_2 :

$$\begin{pmatrix} \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

There are 8 independent components.

- (ii) Group m :

$$\begin{pmatrix} \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

There are 10 independent components.

- (iii) Group $2/m$: all the components are equal to zero.

1.1.4.10.4.3. Orthorhombic system

- (i) Group 222 :

$$\begin{pmatrix} \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

There are 3 independent components.

- (ii) Group $mm2$:

$$\begin{pmatrix} \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

There are 5 independent components.

- (iii) Group mmm : all the components are equal to zero.

1.1.4.10.4.4. Trigonal system

- (i) Group 3:

$$\begin{pmatrix} \bullet & \ominus & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

where the symbol \ominus means that the corresponding component is equal to the opposite of that to which it is linked, \odot means that the component is equal to twice minus the value of the component to which it is linked for d_{ijk} and to minus the value of the component to which it is linked for e_{ijk} . There are 6 independent components.

- (ii) Group 32 , twofold axis parallel to Ox_1 :

$$\begin{pmatrix} \bullet & \ominus & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

with the same conventions. There are 4 independent components.

- (iii) Group $3m$, mirror perpendicular to Ox_1 :

$$\begin{pmatrix} \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

with the same conventions. There are 4 independent components.

- (iv) Groups $\bar{3}$ and $3m$: all the components are equal to zero.

1.1.4.10.4.5. Tetragonal, hexagonal and cylindrical systems

- (i) Groups 4, 6 and A_∞ :

$$\begin{pmatrix} \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

There are 4 independent components.

- (ii) Groups $422, 622$ and $A_\infty \infty A_2$:

$$\begin{pmatrix} \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

There is 1 independent component.

- (iii) Groups $4mm, 6mm$ and $A_\infty \infty M$:

$$\begin{pmatrix} \bullet & \bullet & \bullet & | & \bullet & \bullet \\ \bullet & \bullet & \bullet & | & \bullet & \bullet \end{pmatrix}$$

There are 3 independent components.