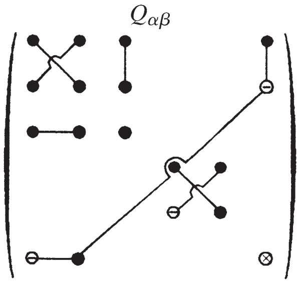
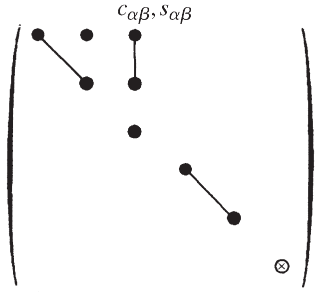
International Tables for Crystallography (2013). Vol. D, Section 1.1.4.10.7.1, pp. 29–30.

$Q_{\alpha\beta}$



8 independent components

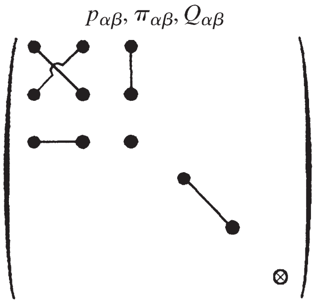
$c_{\alpha\beta}, s_{\alpha\beta}$



5 independent components

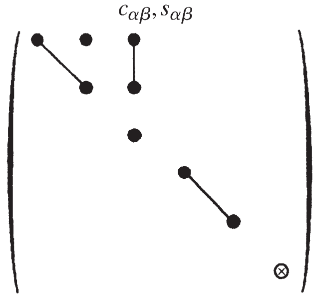
(ii) Groups 622, 6*mm*, $\bar{6}2m$ and 6/*mm*:

$p_{\alpha\beta}, \pi_{\alpha\beta}, Q_{\alpha\beta}$



6 independent components

$c_{\alpha\beta}, s_{\alpha\beta}$

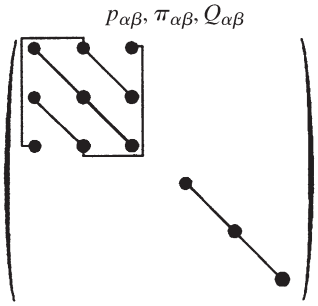


5 independent components

1.1.4.10.6.7. *Cubic system*

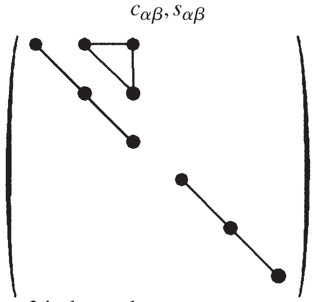
(i) Groups 23 and 3*m*:

$p_{\alpha\beta}, \pi_{\alpha\beta}, Q_{\alpha\beta}$



4 independent components

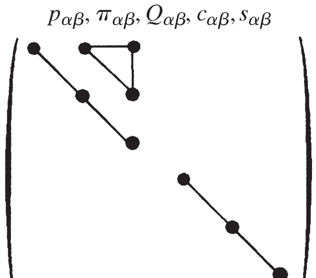
$c_{\alpha\beta}, s_{\alpha\beta}$



3 independent components

(ii) Groups 432, $\bar{4}3m$ and $m\bar{3}m$:

$p_{\alpha\beta}, \pi_{\alpha\beta}, Q_{\alpha\beta}, c_{\alpha\beta}, s_{\alpha\beta}$

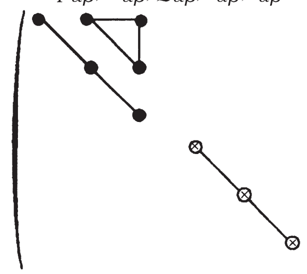


3 independent components

1.1.4.10.6.8. *Spherical system*

For all tensors

$p_{\alpha\beta}, \pi_{\alpha\beta}, Q_{\alpha\beta}, c_{\alpha\beta}, s_{\alpha\beta}$



2 independent components

1.1.4.10.7. *Reduction of the number of independent components of axial tensors of rank 2*

It was shown in Section 1.1.4.5.3.2 that axial tensors of rank 2 are actually tensors of rank 3 antisymmetric with respect to two indices. The matrix of independent components of a tensor such that

$$g_{ijk} = -g_{jik}$$

is given by

$$\left(\begin{array}{ccc|ccc|ccc} & 122 & 133 & 123 & 131 & & 132 & & 121 \\ -121 & & 223 & & 231 & -122 & 232 & -123 & \\ -131 & -232 & & -233 & & -132 & & -133 & -231 \end{array}\right).$$

The second-rank axial tensor g_{kl} associated with this tensor is defined by

$$g_{kl} = \tfrac{1}{2}\varepsilon_{ijk} g_{ijl}.$$

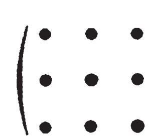
For instance, the piezomagnetic coefficients that give the magnetic moment M_i due to an applied stress T_α are the components of a second-rank axial tensor, $\Lambda_{i\alpha}$ (see Section 1.5.7.1):

$$M_i = \Lambda_{i\alpha} T_\alpha.$$

1.1.4.10.7.1. *Independent components according to the following point groups*

(i) *Triclinic system*

(*a*) Group 1:

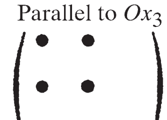


(*b*) Group $\bar{1}$: all components are equal to zero.

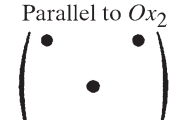
(ii) *Monoclinic system*

(*a*) Group 2:

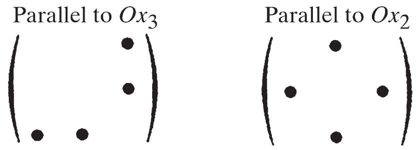
Parallel to Ox_3



Parallel to Ox_2

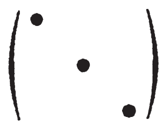


(b) Group m:

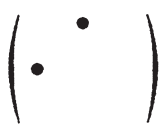


(c) Group $2/m$: all components are equal to zero.

(iii) *Orthorhombic system*
(a) Group 222:

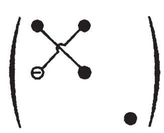


(b) Group $mm2$:

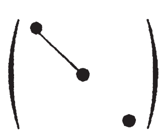


(c) Group mmm: all components are equal to zero.

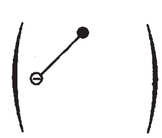
(iv) *Trigonal, tetragonal, hexagonal and cylindrical systems*
(a) Groups 3, 4, 6 and A_∞:



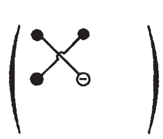
(b) Groups 32, 42, 62 and $A_\infty \infty A_2$:



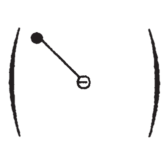
(c) Groups $3m$, $4m$, $6m$ and $A_\infty \infty M$:



(d) Group $\bar{4}$:

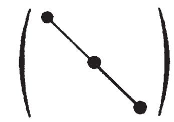


(e) Group $\bar{4}2m$:



(f) Groups $\bar{3}$, $4/m$, $\bar{6}2m$, $\bar{3}m$, $4/mm$ and $6/mm$: all components are equal to zero.

(v) *Cubic and spherical systems*
(a) Groups 23, 432 and ∞A_∞:



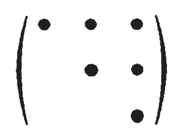
The axial tensor is reduced to a pseudoscalar.
(b) Groups $m\bar{3}$, $\bar{4}3m$, $m\bar{3}m$ and $\infty(A_\infty/M)C$: all components are equal to zero.

1.1.4.10.7.2. *Independent components of symmetric axial tensors according to the following point groups*

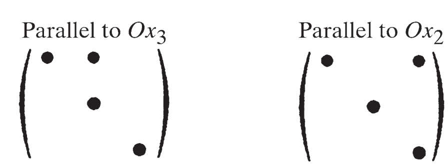
Some axial tensors are also symmetric. For instance, the optical rotatory power of a gyrotropic crystal in a given direction of direction cosines $\alpha_1, \alpha_2, \alpha_3$ is proportional to a quantity G defined by (see Section 1.6.5.4)

$$G = g_{ij}\alpha_i\alpha_j,$$

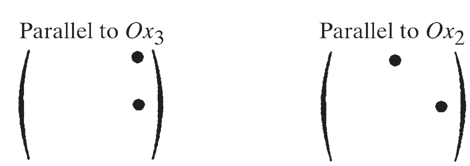
where the gyration tensor g_{ij} is an axial tensor. This expression shows that only the symmetric part of g_{ij} is relevant. This leads to a further reduction of the number of independent components:
(i) *Triclinic system*
(a) Group 1:



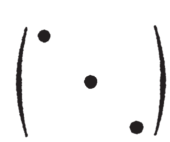
(b) Group $\bar{1}$: all components are equal to zero.
(ii) *Monoclinic system*
(a) Group 2:



(b) Group m:



(c) Group $2/m$: all components are equal to zero.
(iii) *Orthorhombic system*
(a) Group 222:



(b) Group $mm2$:

(c) Group mmm: all components are equal to zero.

references