of mirrors parallel to it, $A_\infty \infty M$. Considered as a cause, the electric field induces for instance the motion of a spherical electric charge parallel to itself. The associated symmetry is the same in each case, and the symmetry of the electric field is identical to that of a force, $A_\infty \infty M$. The electric polarization or the electric displacement have the same symmetry.

1.1.4.3.2. *Symmetry of magnetic induction*

The determination of the symmetry of magnetic quantities is more delicate. Considered as an effect, magnetic induction may be obtained by passing an electric current in a loop (Fig. 1.1.4.2). The corresponding symmetry is that of a cylinder rotating around its axis, $(A_\infty/M)C$. Conversely, the variation of the flux of magnetic induction through a loop induces an electric current in the loop. If the magnetic induction is considered as a cause, its effect has the same symmetry. The symmetry associated with the magnetic induction is therefore $(A_\infty/M)C$.

This symmetry is completely different from that of the electric field. This difference can be understood by reference to Maxwell's equations, which relate electric and magnetic quantities:

$$\text{curl } \mathbf{E} = \nabla \wedge \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}; \quad \text{curl } \mathbf{H} = \nabla \wedge \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}.$$

It was seen in Section 1.1.3.8.3 that the curl is an axial vector because it is a vector product. Maxwell's equations thus show that if the electric quantities ($\mathbf{E}$, $\mathbf{D}$) are polar vectors, the magnetic quantities ($\mathbf{B}$, $\mathbf{H}$) are axial vectors and *vice versa*; the equations of Maxwell are, in effect, perfectly symmetrical on this point. Indeed, one could have been tempted to determine the symmetry of the magnetic field by considering interactions between magnets, which would have led to the symmetry $A_\infty \infty M$ for the magnetic quantities. However, in the world where we live and where the origin of magnetism is in the spin of the electron, the magnetic field is an axial vector of symmetry $(A_\infty/M)C$ while the electric field is a polar vector of symmetry $A_\infty \infty M$.

1.1.4.4. *Superposition of several causes in the same medium – pyroelectricity and piezolectricity*

1.1.4.4.1. *Introduction*

Let us now consider a phenomenon resulting from the superposition of several causes in the same medium. The symmetry of the global cause is the intersection of the groups of symmetry of the various causes: the asymmetries add up (Curie, 1894). This remark can be applied to the determination of the point groups where physical properties such as pyroelectricity or piezoelectricity are possible.

1.1.4.4.2. *Pyroelectricity*

Pyroelectricity is the property presented by certain materials that exhibit electric polarization when the temperature is changed uniformly. Actually, this property appears in crystals for which the centres of gravity of the positive and negative charges do not coincide in the unit cell. They present therefore a spontaneous polarization that varies with temperature because, owing to thermal expansion, the distances between these centres of gravity are temperature dependent. A very important case is that of the ferroelectric crystals where the direction of the polarization can be changed under the application of an external electric field.

From the viewpoint of symmetry, pyroelectricity can be considered as the superposition of two causes, namely the crystal with its symmetry on one hand and the increase of temperature, which is isotropic, on the other. The intersection of the groups of symmetry of the two causes is in this case identical to the group of symmetry of the crystal. The symmetry associated with the effect is that of the electric polarization that is produced, $A_\infty \infty M$. Since the asymmetry of the cause must pre-exist in the causes, the latter may not possess more than one axis of symmetry nor mirrors other than those parallel to the single axis. The only crystal point groups compatible with this condition are

$$1, 2, 3, 4, 6, m, 2mm, 3m, 4mm, 6mm.$$

There are therefore only ten crystallographic groups that are compatible with the pyroelectric effect. For instance, tourmaline, in which the effect was first observed, belongs to $3m$.

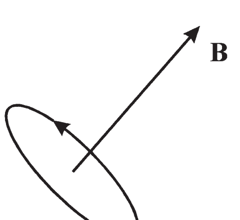

Fig. 1.1.4.2. Symmetry of magnetic induction.

1.1.4.4.3. *Piezoelectricity*

Piezoelectricity, discovered by the Curie brothers (Curie & Curie, 1880), is the property presented by certain materials that exhibit an electric polarization when submitted to an applied mechanical stress such as a uniaxial compression (see, for instance, Cady, 1964; Ikeda, 1990). The converse effect, namely their changes in shape when they are submitted to an external electric field, was predicted by Lippmann (1881) and discovered by J. & P. Curie (Curie & Curie, 1881). The physical interpretation of piezoelectricity is the following: under the action of the applied stress, the centres of gravity of negative and positive charges move to different positions in the unit cell, which produces an electric polarization.

From the viewpoint of symmetry, piezoelectricity can be considered as the superposition of two causes, the crystal with its own symmetry and the applied stress. The symmetry associated with a uniaxial compression is that of two equal and opposite forces, namely $A_\infty/M \;\; \infty A_2/\infty MC$. The effect is an electric polarization, of symmetry $A_\infty \infty M$, which must be higher than or equal to the intersection of the symmetries of the two causes:

$$\frac{A_\infty}{M}\frac{\infty A_2}{\infty M} C \bigcap S_{\rm crystal} \leq A_\infty \infty M,$$

where $S_{\rm crystal}$ denotes the symmetry of the crystal.

It may be noted that the effect does not possess a centre of symmetry. The crystal point groups compatible with the property of piezoelectricity are therefore among the 21 noncentrosymmetric point groups. More elaborate symmetry considerations show further that group 432 is also not compatible with piezoelectricity. This will be proved in Section 1.1.4.10.4 using the symmetry properties of tensors. There are therefore 20 point groups compatible with piezoelectricity:

$$1,\ 2,\ m,\ 222,\ 2mm,$$
$$3,\ 32,\ 3m,\ 4,\ \bar{4},\ 422,\ 4mm,\ \bar{4}2m,\ 6,\ \bar{6},\ 622,\ 6mm,\ \bar{6}2m$$
$$23,\ \bar{4}3m.$$

The intersection of the symmetries of the crystal and of the applied stress depend of course on the orientation of this stress relative to the crystallographic axes. Let us take, for instance, a crystal of quartz, which belongs to group $32 = A_3 3A_2$. The above condition becomes