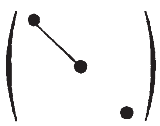
International Tables for Crystallography (2013). Vol. D, Section 1.1.4.8, pp. 17–20.

numerically equal to that to which it is linked, but of opposite sign. There are 3 independent components.

1.1.4.7.4.2. *Groups* $\bar{3}m$, 32, 3*m*; 4/*mm*, 422, 4*mm*, $\bar{4}2m$; 6/*mm*, 622, 6*mm*, $\bar{6}2m$; $(A_\infty/M)\infty(A_2/M)C$, $A_\infty\infty A_2$

The result is obtained by combining the preceding result and that corresponding to a twofold axis normal to the fourfold axis. One finds



There are 2 independent components.

1.1.4.7.5. *Cubic and spherical systems*

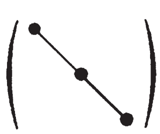
The cubic system is characterized by the presence of threefold axes along the ⟨111⟩ directions. The action of a threefold axis along [111] on the components x_1, x_2, x_3 of a vector results in a permutation of these components, which become, respectively, x_2, x_3, x_1 and then x_3, x_1, x_2. One deduces that the components of a tensor of rank 2 satisfy the relations

$$t_1^1 = t_2^2 = t_3^3.$$

The cubic groups all include as a subgroup the group 23 of which the generating elements are a twofold axis along Ox_3 and a threefold axis along [111]. If one combines the corresponding results, one deduces that

$$t_1^2 = t_2^3 = t_3^1 = t_1^3 = t_2^1 = t_3^2 = 0,$$

which can be summarized by

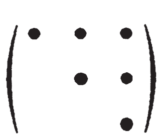


There is a single independent component and the medium behaves like a property represented by a tensor of rank 2, like an isotropic medium.

1.1.4.7.6. *Symmetric tensors of rank 2*

If the tensor is symmetric, the number of independent components is still reduced. One obtains the following, representing the nonzero components for the leading diagonal and for one half of the others.

1.1.4.7.6.1. *Triclinic system*



There are 6 independent components. It is possible to interpret the number of independent components of a tensor of rank 2 by considering the associated quadric, for instance the optical indicatrix. In the triclinic system, the quadric is any quadric. It is characterized by six parameters: the lengths of the three axes and the orientation of these axes relative to the crystallographic axes.

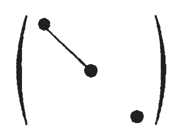
1.1.4.7.6.2. *Monoclinic system (twofold axis parallel to* Ox_2*)*

There are 4 independent components. The quadric is still any quadric, but one of its axes coincides with the twofold axis of the monoclinic lattice. Four parameters are required: the lengths of the axes and one angle.

1.1.4.7.6.3. *Orthorhombic system*

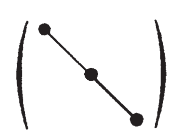
There are 3 independent components. The quadric is any quadric, the axes of which coincide with the crystallographic axes. Only three parameters are required.

1.1.4.7.6.4. *Trigonal, tetragonal and hexagonal systems, isotropic groups*



There are 2 independent components. The quadric is of revolution. It is characterized by two parameters: the lengths of its two axes.

1.1.4.7.6.5. *Cubic system*



There is 1 independent component. The associated quadric is a sphere.

1.1.4.8. *Reduction of the components of a tensor of rank 3*

1.1.4.8.1. *Triclinic system*

1.1.4.8.1.1. *Group* 1

All the components are independent. Their number is equal to 27. They are usually represented as a 3 × 9 matrix which can be subdivided into three 3 × 3 submatrices:

$$\left(\begin{array}{ccc|ccc|ccc} 111 & 122 & 133 & 123 & 131 & 112 & 132 & 113 & 121 \\ 211 & 222 & 233 & 223 & 231 & 212 & 232 & 213 & 221 \\ 311 & 322 & 333 & 323 & 331 & 312 & 332 & 313 & 321 \end{array}\right).$$

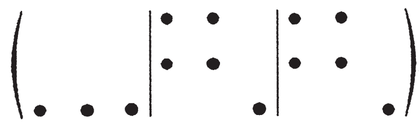
1.1.4.8.1.2. *Group* $\bar{1}$

All the components are equal to zero.

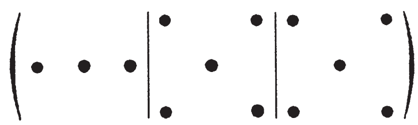
1.1.4.8.2. *Monoclinic system*

1.1.4.8.2.1. *Group* 2

Choosing the twofold axis parallel to Ox_3 and applying the direct inspection method, one finds

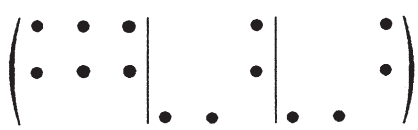


There are 13 independent components. If the twofold axis is parallel to Ox_2, one finds

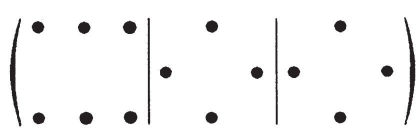


1.1.4.8.2.2. *Group m*

One obtains the matrix representing the operator m by multiplying by -1 the coefficients of the matrix representing a twofold axis. The result of the reduction will then be exactly complementary: the components of the tensor which include an odd number of 3's are now equal to zero. One writes the result as follows:



There are 14 independent components. If the mirror axis is normal to Ox_2, one finds



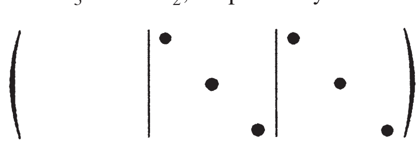
1.1.4.8.2.3. *Group* $2/m$

All the components are equal to zero.

1.1.4.8.3. *Orthorhombic system*

1.1.4.8.3.1. *Group* 222

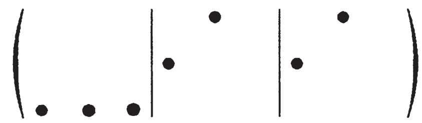
There are three orthonormal twofold axes. The reduction is obtained by combining the results associated with two twofold axes, parallel to Ox_3 and Ox_2, respectively.



There are 6 independent components.

1.1.4.8.3.2. *Group mm*2

The reduction is obtained by combining the results associated with a twofold axis parallel to Ox_3 and with a mirror normal to Ox_2:



There are 7 independent components.

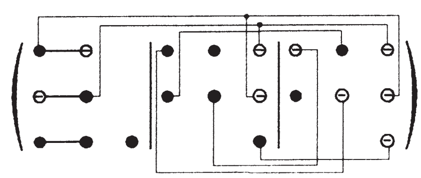
1.1.4.8.3.3. *Group mmm*

All the components are equal to zero.

1.1.4.8.4. *Trigonal system*

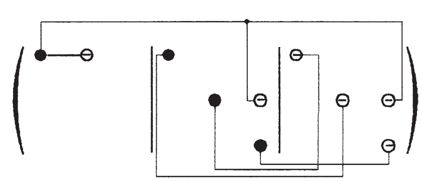
1.1.4.8.4.1. *Group* 3

The threefold axis is parallel to Ox_3. The matrix method should be used here. One finds



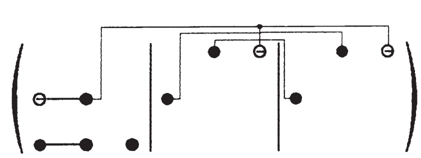
There are 9 independent components.

1.1.4.8.4.2. *Group* 32 *with a twofold axis parallel to* Ox_1



There are 4 independent components.

1.1.4.8.4.3. *Group* 3*m with a mirror normal to* Ox_1



There are 4 independent components.

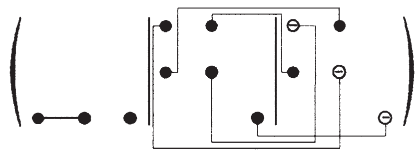
1.1.4.8.4.4. *Groups* $\bar{3}$ *and* $\bar{3}m$

All the components are equal to zero.

1.1.4.8.5. *Tetragonal system*

1.1.4.8.5.1. *Group* 4

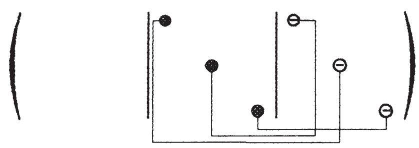
The method of direct inspection can be applied for a fourfold axis. One finds



There are 7 independent components.

1.1.4.8.5.2. *Group* 422

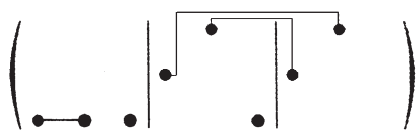
One combines the reductions for groups 4 and 222:



There are 3 independent components.

1.1.4.8.5.3. *Group* 4*mm*

One combines the reductions for groups 4 and 2*m*:



There are 4 independent components.

1.1.4.8.5.4. *Group* $4/m$

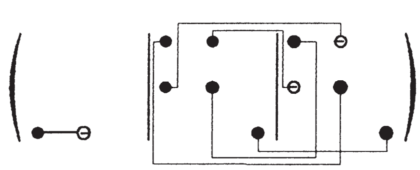
All the components are equal to zero.

1.1.4.8.5.5. *Group* $\bar{4}$

The matrix corresponding to axis $\bar{4}$ is

$$\begin{pmatrix} 0 & \bar{1} & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix}$$

and the form of the 3×9 matrix is

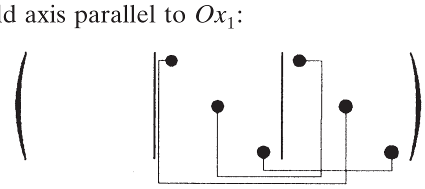


There are 6 independent components.

1.1.4.8.5.6. *Group* $\bar{4}2m$

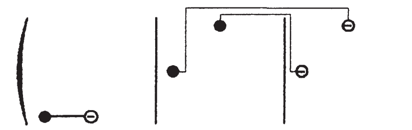
One combines either the reductions for groups $\bar{4}$ and 222, or the reductions for groups $\bar{4}$ and 2*mm*.

(i) Twofold axis parallel to Ox_1:



There are 6 independent components.

(ii) Mirror perpendicular to Ox_1 (the twofold axis is at 45°)



The number of independent components is of course the same, 6.

1.1.4.8.5.7. *Group* $4/mm$

All the components are equal to zero.

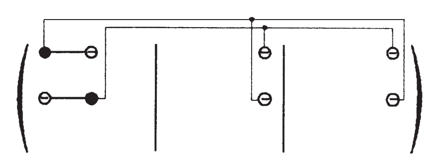
1.1.4.8.6. *Hexagonal and cylindrical systems*

1.1.4.8.6.1. *Groups* 6, A_∞, 622, $A_\infty \infty A_2$, 6*mm and* $A_\infty \infty M$

It was shown in Section 1.1.4.6.2.3 that, in the case of tensors of rank 3, the reduction is the same for axes of order 4, 6 or higher. The reduction will then be the same as for the tetragonal system.

1.1.4.8.6.2. *Group* $\bar{6} = 3/m$

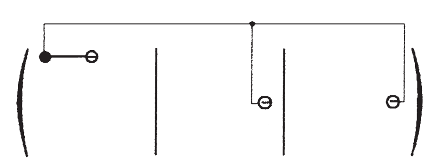
One combines the reductions for the groups corresponding to a threefold axis parallel to Ox_3 and to a mirror perpendicular to Ox_3:



There are 2 independent components.

1.1.4.8.6.3. *Group* $\bar{6}2m$

One combines the reductions for groups 6 and 2*mm*:



There is 1 independent component.

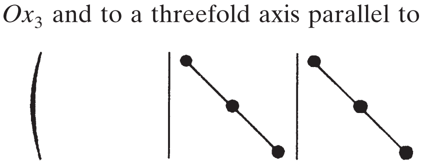
1.1.4.8.6.4. *Groups* $6/m$, $(A_\infty/M)C$, $6/mm$ *and* $(A_\infty/M)\infty(A_2/M)C$

All the components are equal to zero.

1.1.4.8.7. *Cubic and spherical systems*

1.1.4.8.7.1. *Group* 23

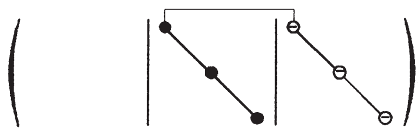
One combines the reductions corresponding to a twofold axis parallel to Ox_3 and to a threefold axis parallel to [111]:



There are 2 independent components.

1.1.4.8.7.2. *Groups* 432 *and* $\infty A_\infty/M$

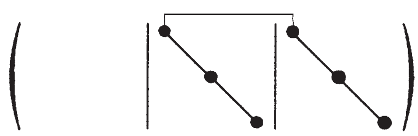
One combines the reductions corresponding to groups 422 and 23:



There is 1 independent component.

1.1.4.8.7.3. *Group* $\bar{4}3m$

One combines the reductions corresponding to groups $\bar{4}2m$ and 23:



There is 1 independent component.

1.1.4.8.7.4. *Groups* $m\bar{3}$, $m\bar{3}m$ *and* $\infty(A_\infty/M)C$

All the components are equal to zero.

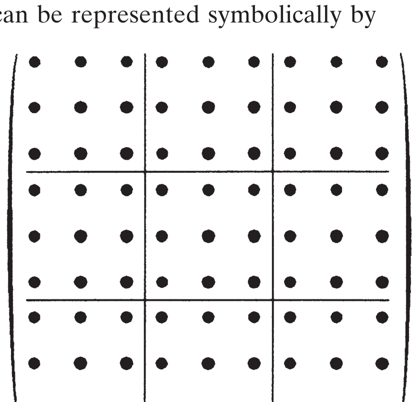
1.1.4.9. *Reduction of the components of a tensor of rank 4*

1.1.4.9.1. *Triclinic system (groups* $\bar{1}$, 1*)*

There is no reduction; all the components are independent. Their number is equal to 81. They are usually represented as a 9×9 matrix, where components t_{ijkl} are replaced by $ijkl$, for brevity:

ij \ *kl*	11	22	33	23	31	12	32	13	21
11	1111	1122	1133	1123	1131	1112	1132	1113	1121
22	2211	2222	2233	2223	2231	2212	2232	2213	2221
33	3311	3322	3333	3323	3331	3312	3332	3313	3321
23	2311	2322	2333	2323	2331	2312	2332	2313	2321
31	3111	3122	3133	3123	3131	3112	3132	3113	3121
12	1211	1222	1233	1223	1231	1212	1232	1213	1221
32	3211	3222	3233	3223	3231	3212	3232	3213	3221
13	1311	1322	1333	1323	1331	1312	1332	1313	1321
21	2111	2122	2133	2123	2131	2112	2132	2113	2121

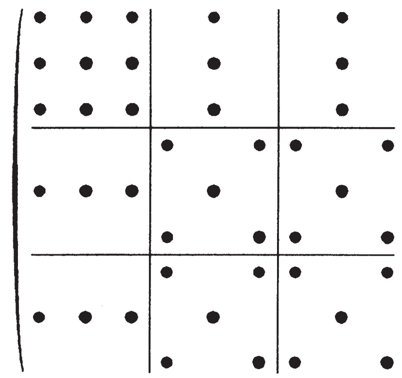
This matrix can be represented symbolically by



where the 9×9 matrix has been subdivided for clarity in to nine 3×3 submatrices.

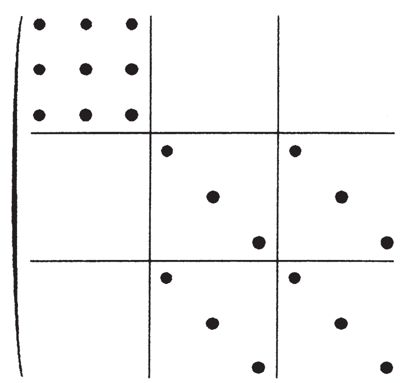
1.1.4.9.2. *Monoclinic system (groups* $2/m$, 2, m*)*

The reduction is obtained by the method of direct inspection. For a twofold axis parallel to Ox_2, one finds



There are 41 independent components.

1.1.4.9.3. *Orthorhombic system (groups mmm,* 2*mm,* 222*)*



There are 21 independent components.

1.1.4.9.4. *Trigonal system*

1.1.4.9.4.1. *Groups* 3 *and* $\bar{3}$

The reduction is first applied in the system of axes tied to the eigenvectors of the operator representing a threefold axis. The system of axes is then changed to a system of orthonormal axes with Ox_3 parallel to the threefold axis:

ij \ *kl*	11	22	33	23	31	12	32	13	21
11	1111	1122	1133	1123	−2231	1112	1132	−2213	1121
22	1122	1111	1133	−1123	2231	−1121	−1132	2213	−1112
33	3311	3311	3333			3312			−3312
23	2311	−2311		2323	2331	1322	2332	2313	1322
31	−3122	3122		3123	3131	3211	3132	3113	3211
12	1211	−2111	1233	2213	1132	1212	2231	1123	1221
32	3211	−3211		3113	−3132	3122	3131	−3123	3122
13	−1322	1322		−2313	2332	2311	−2331	2323	2311
21	2111	−121	−1233	2213	1132	1221	2231	1123	1212

with

$$\left.\begin{aligned} t_{1111} - t_{1122} &= \quad t_{1212} + t_{1221} \\ t_{1112} + t_{1121} &= -(t_{1211} + t_{2111}). \end{aligned}\right\}$$

There are 27 independent components.

references