## 1.1. INTRODUCTION TO THE PROPERTIES OF TENSORS

numerically equal to that to which it is linked, but of opposite sign. There are 3 independent components.

1.1.4.7.4.2. *Groups $\bar{3}m$, 32, 3m; 4/mm, 422, 4mm, $\bar{4}2m$; 6/mm, 622, 6mm, $\bar{6}2m$; $(A_\infty/M)\infty(A_2/M)C$, $A_\infty\infty A_2$*

The result is obtained by combining the preceding result and that corresponding to a twofold axis normal to the fourfold axis. One finds

$$\begin{pmatrix} \bullet\!\!\diagdown\!\!\bullet & \\ & \\ & \bullet \end{pmatrix}$$
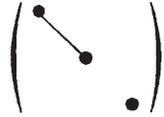
There are 2 independent components.

1.1.4.7.5. *Cubic and spherical systems*

The cubic system is characterized by the presence of threefold axes along the $\langle 111 \rangle$ directions. The action of a threefold axis along [111] on the components x_1, x_2, x_3 of a vector results in a permutation of these components, which become, respectively, x_2, x_3, x_1 and then x_3, x_1, x_2. One deduces that the components of a tensor of rank 2 satisfy the relations

$$t_1^1 = t_2^2 = t_3^3.$$

The cubic groups all include as a subgroup the group 23 of which the generating elements are a twofold axis along Ox_3 and a threefold axis along [111]. If one combines the corresponding results, one deduces that

$$t_1^2 = t_2^3 = t_3^1 = t_1^3 = t_2^1 = t_3^2 = 0,$$

which can be summarized by

$$\begin{pmatrix} \bullet\!\!\diagdown\!\! & \\ & \bullet\!\!\diagdown\!\! \\ & & \bullet \end{pmatrix}$$
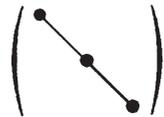
There is a single independent component and the medium behaves like a property represented by a tensor of rank 2, like an isotropic medium.

1.1.4.7.6. *Symmetric tensors of rank 2*

If the tensor is symmetric, the number of independent components is still reduced. One obtains the following, representing the nonzero components for the leading diagonal and for one half of the others.

1.1.4.7.6.1. *Triclinic system*

$$\begin{pmatrix} \bullet & \bullet & \bullet \\ & \bullet & \bullet \\ & & \bullet \end{pmatrix}$$

There are 6 independent components. It is possible to interpret the number of independent components of a tensor of rank 2 by considering the associated quadric, for instance the optical indicatrix. In the triclinic system, the quadric is any quadric. It is characterized by six parameters: the lengths of the three axes and the orientation of these axes relative to the crystallographic axes.

1.1.4.7.6.2. *Monoclinic system (twofold axis parallel to Ox_2)*

$$\begin{pmatrix} \bullet & & \bullet \\ & \bullet & \\ \bullet & & \bullet \end{pmatrix}$$
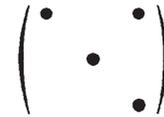
There are 4 independent components. The quadric is still any quadric, but one of its axes coincides with the twofold axis of the monoclinic lattice. Four parameters are required: the lengths of the axes and one angle.

1.1.4.7.6.3. *Orthorhombic system*

$$\begin{pmatrix} \bullet & & \\ & \bullet & \\ & & \bullet \end{pmatrix}$$

There are 3 independent components. The quadric is any quadric, the axes of which coincide with the crystallographic axes. Only three parameters are required.

1.1.4.7.6.4. *Trigonal, tetragonal and hexagonal systems, isotropic groups*

$$\begin{pmatrix} \bullet\!\!\diagdown\!\!\bullet & \\ & \\ & \bullet \end{pmatrix}$$

There are 2 independent components. The quadric is of revolution. It is characterized by two parameters: the lengths of its two axes.

1.1.4.7.6.5. *Cubic system*

$$\begin{pmatrix} \bullet\!\!\diagdown\!\! & \\ & \bullet\!\!\diagdown\!\! \\ & & \bullet \end{pmatrix}$$

There is 1 independent component. The associated quadric is a sphere.

1.1.4.8. *Reduction of the components of a tensor of rank 3*

1.1.4.8.1. *Triclinic system*

1.1.4.8.1.1. *Group 1*

All the components are independent. Their number is equal to 27. They are usually represented as a 3×9 matrix which can be subdivided into three 3×3 submatrices:

$$\begin{pmatrix} 111 & 122 & 133 & 123 & 131 & 112 & 132 & 113 & 121 \\ 211 & 222 & 233 & 223 & 231 & 212 & 232 & 213 & 221 \\ 311 & 322 & 333 & 323 & 331 & 312 & 332 & 313 & 321 \end{pmatrix}.$$

1.1.4.8.1.2. *Group $\bar{1}$*

All the components are equal to zero.

references