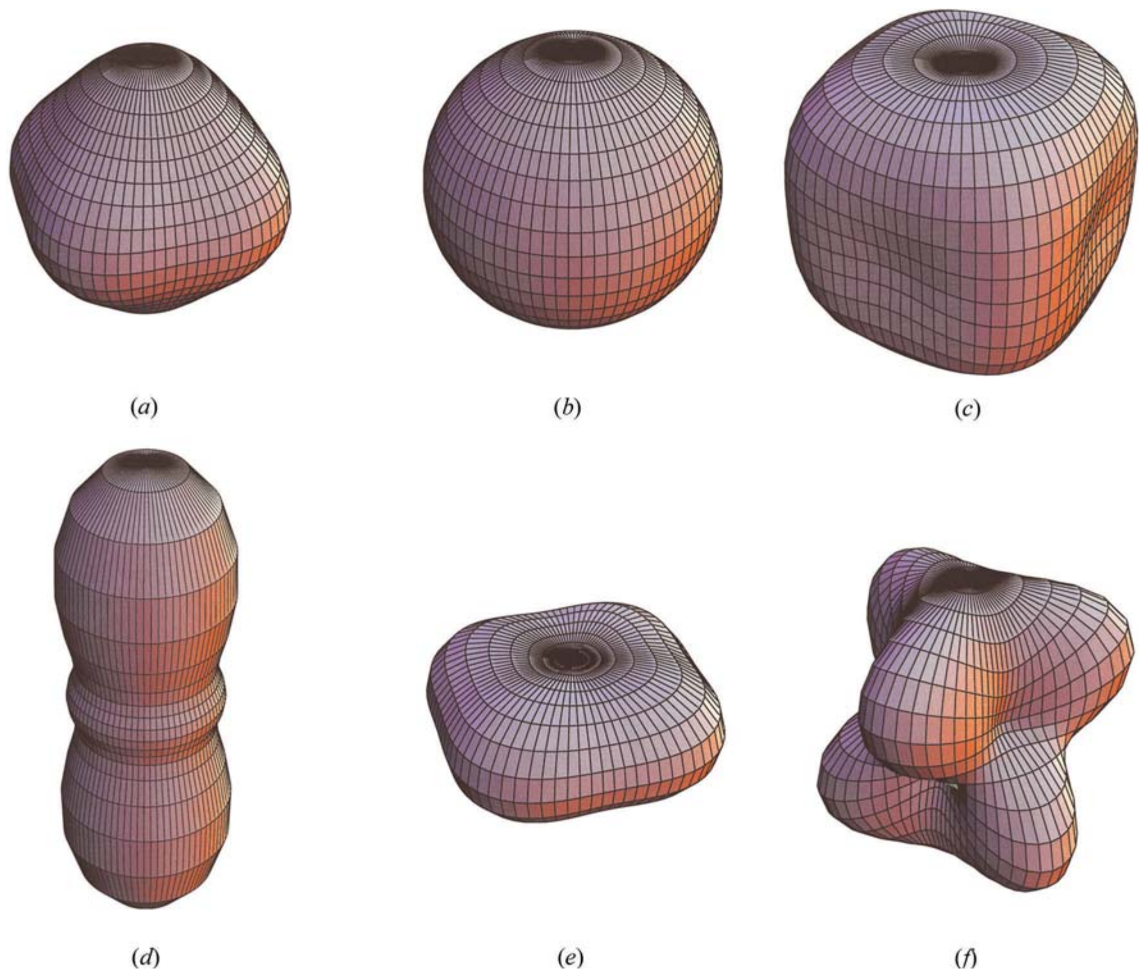

Fig. 1.3.3.4. Representation surface of the inverse of Young's modulus. (*a*) Al, cubic, anisotropy factor > 1; (*b*) W, cubic, anisotropy factor $= 1$; (*c*) NaCl, cubic, anisotropy factor < 1; (*d*) Zn, hexagonal; (*e*) Sn, tetragonal; (*f*) calcite, trigonal.

$$\begin{aligned} T_1 &= c_{11}S_1 + c_{12}(S_2 + S_3) \\ T_2 &= c_{12}S_1 + c_{11}S_2 + c_{12}S_3 \\ T_3 &= c_{12}(S_1 + S_2) + c_{11}S_3. \end{aligned}$$

These relations can equally well be written in the symmetrical form

$$\begin{aligned} T_1 &= (c_{11} - c_{12})S_1 + c_{12}(S_1 + S_2 + S_3) \\ T_2 &= (c_{11} - c_{12})S_2 + c_{12}(S_1 + S_2 + S_3) \\ T_3 &= (c_{11} - c_{12})S_3 + c_{12}(S_1 + S_2 + S_3). \end{aligned}$$

If one introduces the Lamé constants,

$$\begin{aligned} \mu &= (1/2)(c_{11} - c_{12}) = c_{44} \\ \lambda &= c_{12}, \end{aligned}$$

the equations may be written in the form often used in mechanics:

Table 1.3.3.3. *Relations between elastic coefficients in isotropic media*

Coefficient	In terms of μ and λ	In terms of μ and ν	In terms of c_{11} and c_{12}
c_{11}	$2\mu + \lambda$	$2\mu(1-\nu)(1-2\nu)$	c_{11}
c_{12}	λ	$2\mu\nu(1-2\nu)$	c_{12}
$c_{44} = 1/s_{44}$	μ	μ	$(c_{11} - c_{12})/2$
$E = 1/s_{11}$	$\mu(2\mu + 3\lambda)/(\mu + \lambda)$	$2\mu(1+\nu)$	See Section 1.3.3.2.3
s_{12}	$-\lambda/[2\mu(2\mu + 3\lambda)]$	$-\nu/[2\mu(1+\nu)]$	See Section 1.3.3.2.3
κ	$3/(2\mu + 3\lambda)$	$3(1-2\nu)/[2\mu(1+\nu)]$	$3/(c_{11} + 2c_{12})$
$\nu = -s_{12}/s_{11}$	$\lambda/[2(2\mu + 3\lambda)]$	ν	$c_{11}/(c_{11} + c_{12})$

$$\begin{aligned} T_1 &= 2\mu S_1 + \lambda(S_1 + S_2 + S_3) \\ T_2 &= 2\mu S_2 + \lambda(S_1 + S_2 + S_3) \\ T_3 &= 2\mu S_3 + \lambda(S_1 + S_2 + S_3). \end{aligned} \quad (1.3.3.16)$$

Two coefficients suffice to define the elastic properties of an isotropic material, s_{11} and s_{12}, c_{11} and c_{12}, μ and λ, μ and ν, *etc.* Table 1.3.3.3 gives the relations between the more common elastic coefficients.

1.3.3.6. *Equilibrium conditions of elasticity for isotropic media*

We saw in Section 1.3.2.3 that the condition of equilibrium is

$$\partial T_{ij}/\partial x_i + \rho F_j = 0.$$

If we use the relations of elasticity, equation (1.3.3.2), this condition can be rewritten as a condition on the components of the strain tensor:

$$c_{ijkl}\frac{\partial S_{kl}}{\partial x_j} + \rho F_i = 0.$$

Recalling that

$$S_{kl} = \tfrac{1}{2}\left[\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right],$$

the condition becomes a condition on the displacement vector, $\mathbf{u}(\mathbf{r})$:

references