

1.3. ELASTIC PROPERTIES

of $[c_{1122} + c_{1221}]$ on the one hand and from that of c_{1221} on the other hand.

1.3.4.4. Polarization of the elastic waves

The Christoffel determinant has three roots and the Christoffel matrix, being Hermitian with real coefficients, has three real eigenvalues and three orthogonal eigenvectors. The wavevector \mathbf{q} , therefore, encompasses three waves with vibration vectors \mathbf{u}_1 , \mathbf{u}_2 , \mathbf{u}_3 which are perpendicular to one another. In the general case, there is no particular angular relationship between the vibration vectors (or polarization vectors). However, if the latter are parallel to certain symmetry directions in the crystal, one of the vibration vectors is along this direction. The corresponding wave is called longitudinal. The two other waves have their polarization direction perpendicular to the wavevector and are thus transverse. If one of the polarization vectors is almost parallel to the wavevector, which often happens, then one speaks of the vibration as being quasi-longitudinal.

1.3.4.5. Relation between velocity of propagation and elastic stiffnesses

We shall limit ourselves to cubic, hexagonal and tetragonal crystals and consider particular cases.

1.3.4.5.1. Cubic crystals

(i) *The wavevector is parallel to [100]*. The Christoffel determinant reduces to

$$\begin{pmatrix} c_{11} - \rho v^2 & 0 & 0 \\ 0 & c_{44} - \rho v^2 & 0 \\ 0 & 0 & c_{44} - \rho v^2 \end{pmatrix} = 0.$$

The three solutions are given in Table 1.3.4.1. These results are valid for a wave propagating in any direction in an isotropic medium.

(ii) *The wavevector is parallel to [110]*. The direction cosines of the wavevector are $1/\sqrt{2}$, $1/\sqrt{2}$, 0. The Christoffel determinant assumes the form

$$\begin{pmatrix} \frac{1}{2}(c_{11} + c_{44}) - \rho v^2 & \frac{1}{2}(c_{12} + c_{44}) & 0 \\ \frac{1}{2}(c_{12} + c_{44}) & \frac{1}{2}(c_{11} + c_{44}) - \rho v^2 & 0 \\ 0 & 0 & c_{44} - \rho v^2 \end{pmatrix} = 0.$$

The three solutions are given in Table 1.3.4.2.

(iii) *The wavevector is parallel to [111]*. The Christoffel determinant assumes the form

$$\begin{pmatrix} c_{11} + 2c_{44} - \rho v^2 & c_{12} + c_{44} & c_{12} + c_{44} \\ c_{12} + c_{44} & c_{11} + 2c_{44} - \rho v^2 & c_{12} + c_{44} \\ c_{12} + c_{44} & c_{12} + c_{44} & c_{44} - \rho v^2 \end{pmatrix} = 0.$$

The solutions are given in Table 1.3.4.3.

1.3.4.5.2. Hexagonal crystals

In hexagonal crystals, there are five independent elastic stiffnesses, c_{11} , c_{33} , c_{12} , c_{13} , c_{44} and $c_{66} = (c_{11} - c_{12})/2$ (Section 1.1.4.10.4).

(i) *The wavevector is parallel to [001]*. The Christoffel determinant reduces to

$$\begin{pmatrix} c_{44} - \rho v^2 & 0 & 0 \\ 0 & c_{44} - \rho v^2 & 0 \\ 0 & 0 & c_{33} - \rho v^2 \end{pmatrix} = 0.$$

The solutions are given in Table 1.3.4.4.

(ii) *The wavevector is parallel to [100]*. The Christoffel determinant readily reduces to

$$\begin{pmatrix} c_{11} - \rho v^2 & 0 & 0 \\ 0 & c_{66} - \rho v^2 & 0 \\ 0 & 0 & c_{44} - \rho v^2 \end{pmatrix} = 0.$$

The three solutions are given in Table 1.3.4.5.

 1.3.4.5.3. Tetragonal crystals (classes $4mm$, $\bar{4}2m$, $4/mmm$)

In tetragonal crystals, there are six independent elastic stiffnesses, c_{11} , c_{33} , c_{12} , c_{13} , c_{44} and c_{66} (Section 1.1.4.10.4).

(i) *The wavevector is parallel to [001]*. The Christoffel determinant reduces to

Table 1.3.4.1. Velocity of propagation when the wavevector is parallel to [100] (cubic crystals)

Velocity of propagation	Polarization vector	Nature of the wave
$v_{\parallel} = \sqrt{c_{11}/\rho}$	[100]	Longitudinal
$v_{\perp} = \sqrt{c_{44}/\rho}$	[010]	Transverse
$v_{\perp} = \sqrt{c_{44}/\rho}$	Any vector normal to [100]	Transverse

Table 1.3.4.2. Velocity of propagation when the wavevector is parallel to [110] (cubic crystals)

Velocity of propagation	Polarization vector	Nature of the wave
$v_{\parallel} = \sqrt{[c_{44} + \frac{1}{2}(c_{11} + c_{12})]/\rho}$	[110]	Longitudinal
$v_{\perp} = \sqrt{\frac{1}{2}(c_{11} - c_{12})/\rho}$	[$\bar{1}10$]	Transverse
$v_{\perp} = \sqrt{c_{44}/\rho}$	[001]	Transverse

Table 1.3.4.3. Velocity of propagation when the wavevector is parallel to [111] (cubic crystals)

Velocity of propagation	Polarization vector	Nature of the wave
$v_{\parallel} = \sqrt{(c_{11} + 2c_{12} + 4c_{44})/3\rho}$	[111]	Longitudinal
$v_{\perp} = \sqrt{(c_{11} - c_{12} + c_{44})/3\rho}$	Any vector normal to [111]	Transverse

Table 1.3.4.4. Velocity of propagation when the wavevector is parallel to [001] (hexagonal crystals)

Velocity of propagation	Polarization vector	Nature of the wave
$v_{\parallel} = \sqrt{c_{33}/\rho}$	[100]	Longitudinal
$v_{\perp} = \sqrt{c_{44}/\rho}$	Any vector normal to [001]	Transverse

Table 1.3.4.5. Velocity of propagation when the wavevector is parallel to [100] (hexagonal crystals)

Velocity of propagation	Polarization vector	Nature of the wave
$v_{\parallel} = \sqrt{c_{11}/\rho}$	[100]	Longitudinal
$v_{\perp} = \sqrt{c_{66}/\rho}$	[010]	Transverse
$v_{\perp} = \sqrt{c_{44}/\rho}$	[001]	Transverse

Table 1.3.4.6. Velocity of propagation when the wavevector is parallel to [001] (tetragonal crystals)

Velocity of propagation	Polarization vector	Nature of the wave
$v_{\parallel} = \sqrt{c_{33}/\rho}$	[100]	Longitudinal
$v_{\perp} = \sqrt{c_{44}/\rho}$	[010]	Transverse
$v_{\perp} = \sqrt{c_{44}/\rho}$	[001]	Transverse

Table 1.3.4.7. Velocity of propagation when the wavevector is parallel to [100] (tetragonal crystals)

Velocity of propagation	Polarization vector	Nature of the wave
$v_{\parallel} = \sqrt{c_{11}/\rho}$	[100]	Longitudinal
$v_{\perp} = \sqrt{c_{66}/\rho}$	[010]	Transverse
$v_{\perp} = \sqrt{c_{44}/\rho}$	[001]	Transverse