

## 1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

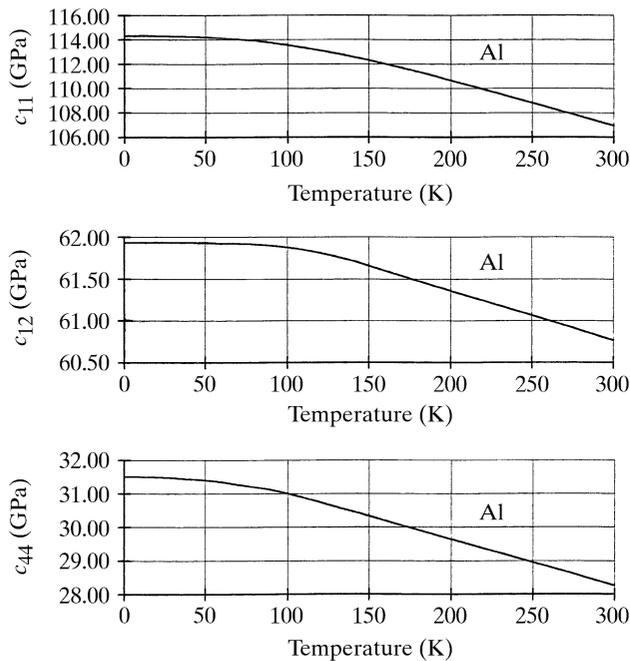


Fig. 1.3.5.1. Temperature dependence of the elastic stiffnesses of an aluminium single crystal (after Every & McCurdy, 1992).

where  $T_{ij}$  represents the stress tensor,  $S_{ij}$  the strain tensor,  $X$  the position of the elementary elements of the solid and  $\Theta$  the temperature.

Different equations of state of solids have been proposed. They correspond to different degrees of approximation that can only be discussed and understood in a microscopic theory of lattice dynamics. The different steps in the development of lattice dynamics, the Einstein model, the Debye model and the Grüneisen model, will be presented in Section 2.1.2.7. Concerning the temperature and the pressure dependences of the elastic constants, we may notice that rather sophisticated models are needed to describe correctly the general trends mentioned above:

(a) In the *Einstein model*, where the  $N$  atoms of a crystal are considered as  $3N$  independent harmonic oscillators, the temperature increase affects only the amplitude of the oscillations and not the average positions of the atoms; consequently, this model can explain neither the thermal expansion nor the *temperature dependence* of the elastic constants. In addition, this theory ignores the difference between isothermal and adiabatic elastic constants. Similarly, if the oscillators are harmonic, the stiffness of the ‘springs’ connecting atoms does not depend on the distances between atoms and the model cannot therefore explain the pressure dependence of the elastic constants, which requires anharmonic ‘springs’ or, more accurately, anharmonic potentials.

(b) In the *Debye model*, the  $3N$  oscillators are not independent but they are still harmonic. The result is that here again the elastic constants are pressure and temperature independent.

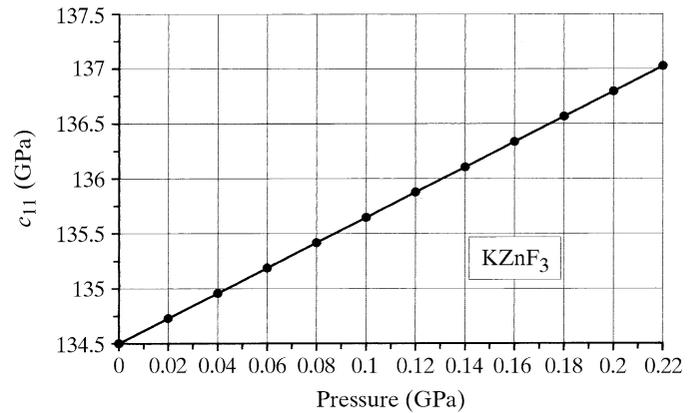


Fig. 1.3.5.2. Pressure dependence of the elastic stiffness  $c_{11}$  of a  $\text{KZnF}_3$  crystal. Reproduced with permission from *Ultrasonics Symposium Proc. IEEE* (Fischer *et al.*, 1980). Copyright (1980) IEEE.

(c) In the *Grüneisen model*, the frequencies of the oscillators are volume-dependent; this so-called ‘quasiharmonic approximation’ can justify the standard behaviour shown in Figs. 1.3.5.1 and 1.3.5.2.

## 1.3.5.2. Temperature dependence of the elastic constants

Table 1.3.5.1 gives typical values of  $(\partial \ln c_{ij}/\partial \Theta)_p$  for some cubic crystals considered within their stability domain. In column 6, the ‘elastic Debye temperature’ of the crystal,  $(\Theta_D)_{el}$ , has been calculated according to the formula

$$(\Theta_D)_{el} = hv/k_B(3n/4\pi)^{1/3},$$

where  $h$  is the Planck constant,  $k_B$  is the Boltzmann constant,  $v$  is an average velocity (see for instance De Launay, 1956) and  $n$  is the number of atoms per unit volume.

It is interesting to compare  $(\Theta_D)_{el}$ , the ‘elastic Debye temperature’, with  $(\Theta_D)_{cal}$ , the ‘calorimetric Debye temperature’. The definition of  $(\Theta_D)_{cal}$  will be given in Section 2.1.2.7. It results from the attempt at founding a universal description for the thermal properties of solids when the temperature is expressed as a reduced temperature,  $\Theta/(\Theta_D)_{cal}$ ;  $(\Theta_D)_{cal}$  is obtained from calorimetric measurements at low temperature. It is worth noting that accurate values of low-temperature elastic constants and low-temperature calorimetric measurements lead to an excellent agreement between  $(\Theta_D)_{el}$  and  $(\Theta_D)_{cal}$  [better than 2 or 3 K (De Launay, 1956)]. This agreement demonstrates the validity of the Debye model in the vicinity of 0 K. From Table 1.3.5.1, we can observe that for ionic crystals  $(\partial \ln c_{11}/\partial \Theta)_p$  is, in general, greater than  $(\partial \ln c_{44}/\partial \Theta)_p$ . This remark is not valid for covalent and metallic crystals. Typical orders of magnitude are given in Table 1.3.5.2. These statements concern only general trends valid for stable crystals.

In the case of *temperature-induced phase transitions*, some elastic constants are softened in the vicinity and sometimes far from the critical temperature. As an example, Fig. 1.3.5.3 shows the temperature dependence of  $c_{44}$  in  $\text{RbCdF}_3$ ,  $\text{CsCdF}_3$  and  $\text{TlCdF}_3$  single crystals.  $\text{RbCdF}_3$  and  $\text{TlCdF}_3$  undergo structural phase transitions at 124 and 191 K, respectively, while  $\text{CsCdF}_3$  remains stable in this temperature range. The softening of  $c_{44}$  when the temperature decreases starts more than 100 K before the critical temperature,  $\Theta_c$ . In contrast, Fig. 1.3.5.4 shows the temperature dependence of  $c_{11}$  in  $\text{KNiF}_3$ , a crystal that undergoes a para-antiferromagnetic phase transition at 246 K; the coupling between the elastic and the magnetic energy is weak, consequently  $c_{11}$  decreases abruptly only a few degrees before the critical temperature. We can generalize this observation and state that the softening of an elastic constant occurs over a large domain of

Table 1.3.5.1. Temperature dependence of the elastic stiffnesses for some cubic crystals

Material	Temperature range (K)	$(\partial \ln c_{11}/\partial \Theta)_p$ ( $10^{-4} \text{ K}^{-1}$ )	$(\partial \ln c_{44}/\partial \Theta)_p$ ( $10^{-4} \text{ K}^{-1}$ )	$(\partial \ln c_{12}/\partial \Theta)_p$ ( $10^{-4} \text{ K}^{-1}$ )	$\Theta_{el}$ (K)
Al	80–300	−3.1	−4.45	−1.3	430
Cu	80–300	−2.01	−3.33	−1.24	344
Ag	50–300	−2.3	−4.0	−1.5	226
Pb	100–300	−4.4	−1.5	−2.8	105
Si	80–300	−0.81	−10.6	−1.10	648
Ge	150–1000	−1.2	−1.15	−1.10	374
ZnS	100–300	−1.2	−0.65	−0.8	347
NaCl	100–300	−7.8	−2.2	−4.7	321
KCl	80–300	−8.3	−2.1	−3.6	236
KBr	80–300	−7.6	−2.1	7	172