Table 1.5.10.1. *Conversion of non-rationalized (except for α) Gaussian units to SI units*

Symbol	Quantity	Gaussian unit and its SI equivalent
$\mathbf{B}$	Magnetic induction	1 gauss (G) = 10^{-4} tesla (T)
$\mathbf{H}$	Magnetic field	1 oersted (Oe) = $10^3/(4\pi)$ A m^{-1}
$\mathbf{M}$	Magnetization (= magnetic moment per unit volume)	1 emu cm^{-3} = 10^3 A m^{-1}
α	Linear magnetoelectric tensor (rationalized units)	1 (dimensionless units) = $4\pi \times 10^{-8}/3$ s m^{-1}
Λ	Piezomagnetic tensor	1 Oe^{-1} = $4\pi \times 10^{-3}$ m A^{-1} = $4\pi \times 10^{-3}$ T Pa^{-1}
χ	Magnetic volume susceptibility	1 (dimensionless units) = 4π (dimensionless units)
χ_g	Magnetic mass susceptibility	1 $cm^3\ g^{-1}$ = $4\pi \times 10^{-3}$ $m^3\ kg^{-1}$
$\chi_{\rm mol}$	Magnetic molar susceptibility	1 $cm^3\ mol^{-1}$ = $4\pi \times 10^{-6}$ $m^3\ mol^{-1}$

In a uniaxial crystal, the magnetostriction in the magnetic field arises mainly as a result of the rotation of the magnetization vector from the direction of the easy axis to the direction of the applied field. The magnetostriction in the magnetic field of an easy-axis hexagonal ferromagnet can be obtained from the relation for the spontaneous magnetostriction (1.5.9.14). In the demagnetized state, such a ferromagnet possesses only two types of antiparallel domains, in which the magnetization is aligned parallel or antiparallel to the hexagonal axis ($n_z = \pm 1$, $n_x = n_y = 0$).

Thus the magnetostriction of the demagnetized state is described by

$$\lambda_\beta^{\rm dem} = h_0 + (h_1 + h_6)\beta_3^2. \qquad (1.5.9.25)$$

The saturation magnetostriction can be calculated for different directions of the applied magnetic field using the equations (1.5.9.14), (1.5.9.15) and (1.5.9.25). If the magnetic field is applied along the x axis ($n_x = 1$, $n_y = n_z = 0$), the saturation magnetostrictions for three directions of the vector $\boldsymbol{\beta}$: $\lambda_\beta^{\rm sat} = \lambda_A, \lambda_B, \lambda_C$ are

$$\begin{aligned} \boldsymbol{\beta} \parallel Ox \quad & \lambda_A = h_2,\\ \boldsymbol{\beta} \parallel Oy \quad & \lambda_B = h_3,\\ \boldsymbol{\beta} \parallel Oz \quad & \lambda_C = -h_1. \end{aligned} \qquad (1.5.9.26)$$

If the magnetic field is applied at an angle of 45° to the hexagonal axis along the [101] direction, the saturation magnetostriction along the magnetic field is described by

$$\lambda_D = \lambda_{101}^{\rm sat} = \tfrac{1}{4}(h_2 - h_1 + 2h_5). \qquad (1.5.9.27)$$

Using the constants λ_A, λ_B, λ_C and λ_D introduced above, the general relation for the magnetostriction caused by magnetization to saturation can be presented in the form

$$\begin{aligned} \lambda_\beta^{\rm sat} &= \lambda_A[(n_1\beta_1 + n_2\beta_2)^2 - (n_1\beta_1 + n_2\beta_2)n_3\beta_3]\\ &\quad + \lambda_B[(1 - n_3^2)(1 - \beta_3^2) - (n_1\beta_1 + n_2\beta_2)^2]\\ &\quad + \lambda_C[(1 - n_3^2)\beta_3^2 - (n_1\beta_1 + n_2\beta_2)n_3\beta_3]\\ &\quad + 4\lambda_D(n_1\beta_1 + n_2\beta_2)n_3\beta_3. \end{aligned} \qquad (1.5.9.28)$$

A typical hexagonal ferromagnet is cobalt. The magnetostriction constants introduced above have the following values for Co at room temperature:

$$\begin{aligned} \lambda_A &= -45 \times 10^{-6}, \quad \lambda_C = +110 \times 10^{-6},\\ \lambda_B &= -95 \times 10^{-6}, \quad \lambda_D = -100 \times 10^{-6}. \end{aligned}$$

A more sophisticated treatment of the symmetry of the magnetostriction constants is given in the monograph of Birss (1964) and in Zalessky (1981).

1.5.9.3. *The difference between the magnetic anisotropies at zero strain and zero stress*

The spontaneous magnetostriction makes a contribution to the magnetic anisotropy (especially in crystals with a cubic prototype). Therefore, to find the full expression for the anisotropy energy, one has to sum up the magnetic U_a^0 [see (1.5.9.5)], the magnetoelastic $U_{\rm me}$ [see (1.5.9.3)] and the elastic $U_{\rm el}$ [see (1.5.9.6)] energies. At zero strain ($S_{ij}^* = 0$), only $U_a^0 \neq 0$. At zero stress

$$\begin{aligned} U_a^0 + U_{\rm me} + U_{\rm el} &= U_a^0 + V_{ij}^0 S_{ij}^* + \tfrac{1}{2} c_{ijk\ell} S_{ij}^* S_{k\ell}^*\\ &= U_a^0 + \tfrac{1}{2} V_{ij}^0 S_{ij}^*. \end{aligned} \qquad (1.5.9.29)$$

Here we used the modified equation (1.5.9.7):

$$\tfrac{1}{2} c_{ijk\ell} S_{ij}^* S_{k\ell}^* = -\tfrac{1}{2} V_{ij}^0 S_{ij}^*. \qquad (1.5.9.30)$$

Substituting the values for the spontaneous magnetostriction, the final equation for the anisotropy energy measured at atmospheric pressure may be written as

$$\begin{aligned} U_a &= U_a^0 + \tfrac{1}{2} V_{ij}^0 S_{ij}^*\\ &= (K_{ij}^0 + K_{ij}')n_i n_j + (K_{ijk\ell}^0 + K_{ijk\ell}')n_i n_j n_k n_\ell\\ &\quad + (K_{ijk\ell mn}^0 + K_{ijk\ell mn}')n_i n_j n_k n_\ell n_m n_n + \ldots. \end{aligned} \qquad (1.5.9.31)$$

As an example, for the ferromagnets with a cubic prototype this equation may be written as

$$U_a = (K_1^0 + K_1')S(n_1^2 n_2^2) + (K_2^0 + K_2')n_1^2 n_2^2 n_3^2. \qquad (1.5.9.32)$$

The coefficients K_1' and K_2' may be expressed in terms of the saturation magnetostriction constants $h_0, \ldots, h_5$ [see (1.5.9.12)] and the elastic stiffnesses $c_{\alpha\beta}$:

$$\begin{aligned} K_1' &= c_{11}[h_0(2h_4 - 3h_3) + h_1(h_1 - h_3 + 3h_4) - h_4(h_3 - 2h_4)]\\ &\quad + c_{12}[2h_0(2h_4 - 3h_3) - (h_1 + h_4)(h_1 + 2h_3)] - \tfrac{1}{2} c_{44} h_2^2, \end{aligned} \qquad (1.5.9.33)$$

$$\begin{aligned} K_2' &= -c_{11}[3h_4(h_1 + h_3) + (h_4 - h_3)(4h_4 - 3h_3)]\\ &\quad + c_{12}[3h_4(h_1 + h_3) + h_3(5h_4 - 6h_3)]\\ &\quad - \tfrac{1}{2} c_{44}(6h_2 + h_5)h_5. \end{aligned} \qquad (1.5.9.34)$$

For cubic crystals, K_i^0 and K_i' are of the same magnitude. As an example, for Ni one has K_1^0 = 80 000 erg cm^{-3} = 8000 J m^{-3} and K_1' = −139 000 erg cm^{-3} = −13 900 J m^{-3}.

1.5.10. Connection between Gaussian and SI units

Numerical values of magnetic quantities are given in the tables and figures in this chapter in Gaussian units together with information on how the corresponding values in SI units are obtained. As a summary, Table 1.5.10.1 gives for each such quantity the corresponding Gaussian unit and its value expressed in SI units. More details on the transformation between Gaussian and SI units are given *e.g.* in the Appendix of Jackson (1999).

references