

## 1.5. MAGNETIC PROPERTIES

and  $\Gamma_3$ , which is  $\Gamma_4$ . Therefore, the ferroelectric polarization is only allowed along the  $c$  axis, as is experimentally observed.

Magnetically induced ferroelectricity is also possible for commensurate structures. Examples include some of the phases in the  $RMn_2O_5$  series (Hur *et al.*, 2004, Chapon *et al.*, 2006), and the ‘up-up-down-down’ spin ordering in the  $MnO_2$  planes (called  $E$ -type ordering) in the orthorhombic  $RMnO_3$  series (Lorenz *et al.*, 2007), where  $R$  = a rare-earth metal. The  $E$ -type magnetic structure is a good example of how a single two-dimensional irreducible representation can induce ferroelectricity. There are also magnetically induced ferromagnetic ferroelectrics, where ferroelectricity arises from antiferromagnetic order, and weak ferromagnetism is present due to uniform canting of the magnetic moments. Here examples include  $CoCr_2O_4$  (Yamaski *et al.*, 2006) and  $Mn_2GeO_4$  (White *et al.*, 2012).

Since 2003, a growing number of magnetically induced ferroelectrics have been discovered. Reviews of their symmetry properties have been given by Harris (2007) and Radaelli & Chapon (2007). This phenomenon has been observed for various different transition-metal ions, and for very different crystal structures. They all have in common the fact that ferroelectricity emerges with magnetic order or with a change of an already existing magnetic order. Competing magnetic interactions and low-dimensional magnetic topologies appear to be beneficial for magnetically induced ferroelectricity. The size of the ferroelectric polarization is orders of magnitude smaller than observed in  $BiFeO_3$  and  $YMnO_3$ .

## 1.5.9. Magnetostriction

The transition to an ordered magnetic state is accompanied by a spontaneous distortion of the lattice, which is denoted spontaneous magnetostriction. The lattice distortion may be specified by the deformation (strain) components  $S_{ij}$ . The undeformed state is defined as the crystal structure that would be realized if the crystal remained in the paramagnetic state at the given temperature. This means that it is necessary to separate the magnetostrictive deformation from the ordinary thermal expansion of the crystal. This can be done by measurements of the magnetostriction in external magnetic fields applied in different directions (see Section 1.5.9.2). The magnetostriction arises because the first derivatives of the exchange and relativistic energies responsible for the magnetic order do not vanish at  $S_{ij} = 0$ . Thus these energies depend linearly on the deformations around  $S_{ij} = 0$ . That part of the magnetic energy which depends on the deformations (and consequently on the stresses) is called the magnetoelastic energy,  $U_{me}$ . To find the equilibrium values of the spontaneous magnetostriction, one also has to take the elastic energy into account.

The magnetoelastic energy includes both an exchange and a relativistic part. In some ferromagnets that are cubic in the paramagnetic phase, the exchange interaction does not lower the cubic symmetry. Thus the exchange part of  $U_{me}$  satisfies the relations

$$\partial U_{me} / \partial S_{ii} = B'_0 \quad \text{and} \quad \partial U_{me} / \partial S_{ij} = 0 \quad (i \neq j). \quad (1.5.9.1)$$

Such a form of the magnetoelastic energy gives rise to an isotropic spontaneous magnetostriction or volume change (volume striction) which does not depend on the direction of magnetization. In what follows, we shall analyse mainly the anisotropic magnetostriction.

The spontaneous magnetostriction deformations are so small (about  $10^{-5}$ ) for some ferro- and antiferromagnets that they cannot be observed by the usual X-ray techniques. However, in materials with ions possessing strong spin-orbit interactions (like  $Co^{2+}$ ), it may be as large as  $10^{-4}$ . The magnetostriction in rare-earth metals and their compounds with iron and cobalt are especially large (up to  $10^{-3}$ ).

Magnetostriction is observed experimentally as a change  $\delta l$  of the linear dimension along a direction specified by a unit vector  $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)$ :

$$\lambda_{\beta} = \delta l / l = \sum_{ij} S_{ij} \beta_i \beta_j, \quad (1.5.9.2)$$

where  $S_{ij}$  are the deformation components, which are functions of the components of the unit vector  $\mathbf{n}$  aligned in the direction of the magnetization. Only the symmetric part of the deformation tensor  $S_{ij}$  has been taken into account, because the antisymmetric part represents a rotation of the crystal as a whole.

The magnetostriction that arises in an applied magnetic field will be discussed in Section 1.5.9.2; Section 1.5.9.1 is devoted to the spontaneous magnetostriction.

## 1.5.9.1. Spontaneous magnetostriction

In this section, we shall assume that the crystal under consideration undergoes a phase transition from the paramagnetic state into a magnetically ordered state. The latter is a single-domain state with the magnetization (or the antiferromagnetic vector) aligned along the vector  $\mathbf{n}$ . As was mentioned above, to solve the problem of the spontaneous magnetostriction we have to minimize the sum of magnetoelastic and elastic energy.

Like the anisotropy energy, the anisotropic part of the magnetoelastic energy can be represented as a series in the components of the unit vector  $\mathbf{n}$ :

$$U_{me} = Q_{k\ell mn} S_{k\ell} n_m n_n + Q_{k\ell mnop} S_{k\ell} n_m n_n n_o n_p + \dots = V_{k\ell}^0 S_{k\ell}. \quad (1.5.9.3)$$

As for every magnetically ordered crystal, this relation contains only even powers of the magnetization unit vector. The components of the tensors  $\mathbf{Q}$  are called magnetostrictive or magnetoelastic coefficients. They are proportional to even powers of the magnetization  $M$  ( $Q_{k\ell mn} \propto M^2$  and  $Q_{k\ell mnop} \propto M^4$ ). The symmetry of the tensors  $\mathbf{Q}_{k\ell mn}$  and  $\mathbf{Q}_{k\ell mnop}$  is defined by the crystallographic point group of the initial paramagnetic phase of the crystal.

It is convenient to consider the magnetoelastic energy as part of a general expansion of the free energy of a crystal into a series with respect to the deformation (as the magnetostrictive deformations are small):

$$V = V^0 + V_{k\ell}^0 S_{k\ell} + \frac{1}{2} V_{k\ell mn}^0 S_{k\ell} S_{mn} + \dots, \quad (1.5.9.4)$$

where all the expansion coefficients  $V^0$  are functions of the components of the magnetization unit vector  $\mathbf{n}$ . The superscripts zero indicate that the expansion coefficients have been calculated relative to the undistorted lattice. Such a state in which, at a given temperature, there is no magnetic interaction to distort the crystal is not realizable practically. It will be shown below that the values of the coefficients  $V_{k\ell}^0$  may be obtained experimentally by observing the magnetostriction in a magnetic field (see Section 1.5.9.2).

The first term in (1.5.9.4) is the anisotropy energy at zero deformation  $U_a^0$ :

$$V^0 = U_a^0 = K_{ij}^0 n_i n_j + K_{ijk\ell}^0 n_i n_j n_k n_\ell + K_{ijk\ell mn}^0 n_i n_j n_k n_\ell n_m n_n. \quad (1.5.9.5)$$

This expression has to be compared with the expression for the anisotropy at zero stress introduced in Section 1.5.3.2 [see (1.5.3.5)]. It is obvious that symmetry imposes the same restrictions on the tensors  $\mathbf{K}$  in both expressions for the anisotropy. Later, we shall discuss these two relations for the anisotropy in more detail.

The second term in (1.5.9.4) is the magnetoelastic energy density, which is displayed in equation (1.5.9.3) and represents the energy of anisotropic deformation.

# 1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

Table 1.5.9.1. Correspondence between matrix indices  $\alpha$ ,  $A$  and tensor indices of the tensors describing spontaneous magnetostriction

$\alpha$	$ij$	$A$	$k\ell mn$
1	11	01	1111
2	22	02	2222
3	33	03	3333
4	23, 32	04	2233, 2323, 2332, 3223, 3232, 3322
5	31, 13	05	3311, 3131, 3113, 1331, 1313, 1133
6	12, 21	06	1122, 1212, 1221, 2112, 2121, 2211
		07	1123, 1132, 1213, 1231, 1312, 1321, 2113, 2131, 2311, 3112, 3121, 3211
		08	2231, 2213, 2321, 2312, 2123, 2132, 3221, 3212, 3122, 1223, 1232, 1322
		09	3312, 3321, 3132, 3123, 3231, 3213, 1332, 1323, 1233, 2331, 2313, 2133
		10	2223, 2232, 2322, 3222
		11	3331, 3313, 3133, 1333
		12	1112, 1121, 1211, 2111
		13	3332, 3323, 3233, 2333
		14	1113, 1131, 1311, 3111
		15	2221, 2212, 2122, 1222

The third term in (1.5.9.4) is quadratic in  $S_{k\ell}$  and can be considered as an additional contribution to the elastic energy arising from the distortion of the lattice by spontaneous magnetostriction. This term is small compared with the main part of the elastic energy, and the effect it produces is called a morphic effect and is usually neglected.

The equilibrium deformation components  $S_{ij}^*$  may be found by minimization of the sum of the magnetoelastic and elastic energies. The latter,  $U_{el}$ , is given by

$$U_{el} = \frac{1}{2} c_{ijkl} S_{ij} S_{kl}, \quad (1.5.9.6)$$

where  $c_{ijkl}$  are the elastic stiffnesses. The minimization leads to

$$\partial(U_{el} + U_{me})/\partial S_{ij} = c_{ijkl} S_{kl}^* + V_{ij}^0 = 0. \quad (1.5.9.7)$$

We shall replace the elastic stiffnesses  $c_{ijkl}$  in this equation by the elastic compliances  $s_{ijkl}$ , taking into account that Hooke's law can be written in two forms (see Section 1.3.3):

$$T_{ij} = c_{ijkl} S_{kl} \quad \text{or} \quad S_{ij} = s_{ijkl} T_{kl}. \quad (1.5.9.8)$$

Thus the relation for the equilibrium components of the strain  $S_{ij}^*$  becomes

$$S_{ij}^* = -s_{ijkl} V_{kl}^0. \quad (1.5.9.9)$$

Combining the relations (1.5.9.9) and (1.5.9.3), we get the following equation for the magnetostrictive strain components  $S_{ij}$  as a function of the magnitude  $M_s$  and direction  $\mathbf{n} = \mathbf{M}_s/M_s$  of the magnetization  $\mathbf{M}_s$ :

$$\begin{aligned} S_{ij}^* &= -s_{ijkl} (\mathcal{Q}_{k\ell mn} n_m n_n + \mathcal{Q}_{k\ell mnop} n_m n_n n_o n_p + \dots) \\ &= M_s^2 N_{ijkl} n_k n_\ell + M_s^4 N_{ijklmn} n_k n_\ell n_m n_n + \dots \end{aligned} \quad (1.5.9.10)$$

Let us denote the spontaneous magnetostriction by  $\lambda_\beta^0$  ( $\beta$  defines the direction of the magnetostriction relative to the crystallographic axes). According to (1.5.9.2), we obtain

$$\lambda_\beta^0 = M_s^2 N_{ijkl} \beta_i \beta_j n_k n_\ell + M_s^4 N_{ijklmn} \beta_i \beta_j n_k n_\ell n_m n_n. \quad (1.5.9.11)$$

Relation (1.5.9.11) shows that  $N_{ijklmn}$  can be chosen as symmetric in its first two indices and symmetric in its last four indices. It can therefore be represented by a  $6 \times 15$  matrix  $N_{\alpha A}$ , where  $\alpha = 1, \dots, 6$  and  $A = 01, \dots, 15$ . Table 1.5.9.1 lists the pairs  $ij$  that correspond to  $\alpha$  and the quadruples  $k\ell mn$  that correspond to  $A$ .

Similarly,  $N_{ijk\ell}$  can be chosen as symmetric in its first two and in its last two indices. It can therefore be represented by a  $6 \times 6$

matrix  $N_{\alpha\beta}$ , where  $\alpha, \beta = 1, \dots, 6$ . The correspondence between the numbers 1 to 6 and pairs  $ij$  or  $k\ell$  is given in Table 1.5.9.1.

The tensors  $N_{ijk\ell}$  and  $N_{ijklmn}$  must satisfy the symmetry of the paramagnetic state of the crystal under consideration. In the case of cubic crystals with fourfold axes (paramagnetic point groups  $4321'$ ,  $\bar{4}3m1'$  or  $m\bar{3}m1'$ ), the two matrices  $N_{\alpha\beta}$  and  $N_{\alpha A}$  possess instead of the 36 and 90 independent components only 3 and 6, i.e.  $N_{11}, N_{12}, N_{44}$  and  $N_{101}, N_{102}, N_{104}, N_{105}, N_{407}, N_{410}$ , respectively. The exact form of the two matrices will be given in the following.

(a) *Cubic crystals.*

If the point group of the paramagnetic crystal is  $4321'$ ,  $\bar{4}3m1'$  or  $m\bar{3}m1'$ , it follows from the Neumann principle that the only nonvanishing components of  $N_{\alpha\beta}$  are  $N_{11} = N_{22} = N_{33}, N_{12} = N_{23} = N_{31} = N_{21} = N_{32} = N_{13}$  and  $N_{44} = N_{55} = N_{66}$ . Similarly, the only nonvanishing components of  $N_{\alpha A}$  are  $N_{101} = N_{202} = N_{303}, N_{102} = N_{203} = N_{301} = N_{103} = N_{201} = N_{302}, N_{104} = N_{205} = N_{306}, N_{105} = N_{206} = N_{304} = N_{106} = N_{204} = N_{305}, N_{407} = N_{508} = N_{609}, N_{410} = N_{511} = N_{612} = N_{413} = N_{514} = N_{615}$ . The spontaneous magnetostriction (1.5.9.11) can then be written as

$$\begin{aligned} \lambda_\beta^0 &= h_0 + h_1 S(n_1^2 \beta_1^2) + 2h_2 S(n_1 n_2 \beta_1 \beta_2) + h_3 S(n_1^2 n_2^2) \\ &\quad + h_4 S(n_1^4 \beta_1^2 + \frac{2}{3} n_1^2 n_2^2) + 2h_5 S(n_1 n_2 n_3^2 \beta_1 \beta_2). \end{aligned} \quad (1.5.9.12)$$

Here an operator  $S()$  has been introduced, which denotes the sum of the three quantities obtained by cyclic permutation of the suffixes in the expression within the brackets. For example,  $S(n_1^2 n_2 n_3 \beta_2 \beta_3) = n_1^2 n_2 n_3 \beta_2 \beta_3 + n_2^2 n_3 n_1 \beta_3 \beta_1 + n_3^2 n_1 n_2 \beta_1 \beta_2$ .

The coefficients  $h_i$  are related in the following way to the components of the matrices  $N_{\alpha\beta}$  and  $N_{\alpha A}$  and the spontaneous magnetization  $M_s$ :

$$\begin{aligned} h_0 &= N_{12} M_s^2 + N_{102} M_s^4, \\ h_1 &= (N_{11} - N_{12}) M_s^2 - 6(N_{104} - N_{105}) M_s^4, \\ h_2 &= 2N_{44} M_s^2 + 4N_{410} M_s^4, \\ h_3 &= [-\frac{2}{3}(N_{101} + 2N_{102}) + 2(N_{104} + 2N_{105})] M_s^4, \\ h_4 &= [N_{101} - N_{102} + 6(N_{104} - N_{105})] M_s^4, \\ h_5 &= 4(3N_{407} - N_{410}) M_s^4. \end{aligned} \quad (1.5.9.13)$$

(b) *Hexagonal crystals.*

The equation for the spontaneous magnetostriction of a crystal that, in its paramagnetic state, has a point group  $6221'$ ,  $6mm1'$ ,  $\bar{6}m21'$  or  $6/mmm1'$ , is of the following form [if we restrict ourselves to the quadratic terms in (1.5.9.11)]:

$$\begin{aligned} \lambda_\beta^0 &= h_0 + h_1 n_3^2 \beta_3^2 + h_2 (n_1^2 \beta_1^2 + n_2^2 \beta_2^2) + h_3 (n_1^2 \beta_2^2 + n_2^2 \beta_1^2) \\ &\quad + 2h_4 n_1 n_2 \beta_1 \beta_2 + 2h_5 n_3 \beta_3 (n_1 \beta_1 + n_2 \beta_2) + h_6 \beta_3^2. \end{aligned} \quad (1.5.9.14)$$

The coefficients  $h_i$  are related to the components  $N_{\alpha\beta}$  and the spontaneous magnetization as follows:

$$\begin{aligned} h_0 &= N_{13} M_s^2 \\ h_1 &= (N_{33} - N_{31}) M_s^2 \\ h_2 &= (N_{11} - N_{13}) M_s^2 \\ h_3 &= (N_{12} - N_{13}) M_s^2 \\ h_4 &= (N_{11} - N_{12}) M_s^2 \\ h_5 &= 2N_{44} M_s^2 \\ h_6 &= (N_{31} - N_{13}) M_s^2 \end{aligned} \quad (1.5.9.15)$$

As mentioned above, the values of the magnetostrictive coefficients  $h_i$  and the spontaneous magnetostriction  $\lambda_\beta^0$  may be obtained from measurements of magnetostriction in a magnetic field. The latter will be discussed in the next section.

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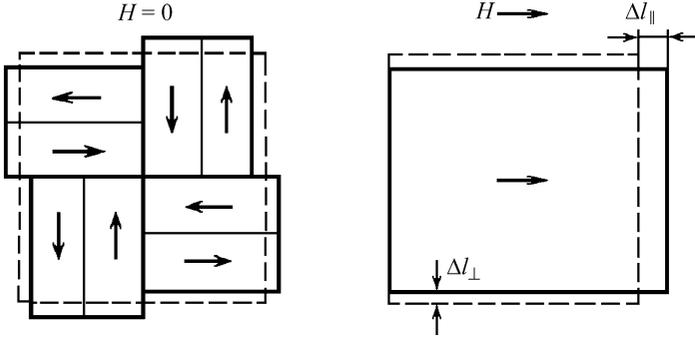


Fig. 1.5.9.1. Diagram explaining the occurrence of magnetostrictive strains in the demagnetized and saturated states of a cube-shaped crystal with a cubic prototype.

Notice that there is some disagreement between our results (1.5.9.12)–(1.5.9.13) and the corresponding results of Mason (1951, 1952), and similarly between (1.5.9.14)–(1.5.9.15) and the results of Mason (1954).

### 1.5.9.2. Magnetostriction in an external magnetic field

There are three reasons for the magnetostriction arising in a magnetic field: (a) the transfer of the crystal into a single-domain state if the magnetic field is directed along one of the easy axes; (b) the deflection of the magnetization (or antiferromagnetic vector) by the magnetic field from the easy axis in a single-domain crystal; (c) the change of the magnetization in a sufficiently strong magnetic field.

Let us begin with case (a) and consider a crystal with cubic symmetry in the paramagnetic state (*i.e.* with a cubic prototype). We calculate the magnetostriction that occurs when the applied magnetic field transforms the crystal from the demagnetized multidomain state into the saturated single-domain state. This transformation is shown schematically in Fig. 1.5.9.1.

Each domain in the demagnetized state is distorted by spontaneous magnetostriction. The number of domains in the sample is usually much larger than shown in the figure. Thus a sample of a crystal with a cubic prototype which in the paramagnetic state has the form of a cube will retain this form in the ordered state. Its linear dimension will be changed as a result of magnetostriction. Averaging these strains over all the domains, one gets the spontaneous magnetostrictive change of the linear dimension of the sample, which is equal for any direction  $x$ ,  $y$  or  $z$ :

$$(\delta l)_{\text{dem}}/l_0 = \lambda^{\text{dem}} = \overline{\lambda_{\beta}^0(n_k)}, \quad (1.5.9.16)$$

where  $n_k$  defines the directions parallel to all the easy axes of the crystal. For crystals with a cubic prototype, there are two principal ordered states: with the easy axis along the  $\langle 111 \rangle$  directions as in nickel or along the  $\langle 100 \rangle$  directions as in iron. Averaging the strains of all eight possible easy-axis directions of the domains in the  $\langle 111 \rangle$ -type ferromagnet we obtain from (1.5.9.12) the following expression for the spontaneous magnetostriction of the demagnetized crystal:

$$\lambda^{\text{dem}} = h_0 + \frac{1}{3}(h_1 + h_3 + h_4). \quad (1.5.9.17)$$

In the case of the  $\langle 100 \rangle$ -type ferromagnet, the averaging over the six groups of domains leads to

$$\lambda^{\text{dem}} = h_0 + \frac{1}{3}(h_1 + h_4). \quad (1.5.9.18)$$

In the saturated state, the sample loses its cubic form. It becomes longer parallel to the magnetic field and thinner perpendicular to it. By definition, the demagnetized state is taken as a reference state for the magnetostriction in the magnetic field. Subtracting from the general relation for spontaneous magne-

Table 1.5.9.2. Magnetostriction data for ferromagnets with prototype symmetry  $m\bar{3}m1'$

Compound	$\lambda_{100} \times 10^6$	$\lambda_{111} \times 10^6$	References†
Fe	20.7	-21.2	(1)
Ni	-45.9	-24.3	(1)
Fe <sub>3</sub> O <sub>4</sub>	-20	78	(2)
YIG ( $T = 300$ K)	-1.4	-2.4	(3)
DyIG ( $T = 300$ K)	-12.5	-5.9	(3)
DyIG ( $T = 4.2$ K)	-1400	-550	(4)

† References: (1) Lee (1955); (2) Bickford *et al.* (1955); (3) Iida (1967); (4) Clark *et al.* (1966).

tostriction (1.5.9.12) the expressions (1.5.9.17) and (1.5.9.18) for the demagnetized sample, Becker & Döring (1939) obtained the equations that describe the anisotropy of the magnetostriction caused by saturation magnetization of the  $\langle 111 \rangle$  and  $\langle 100 \rangle$  types of magnetic crystals:

$\langle 111 \rangle$  type:

$$\begin{aligned} \lambda_{\beta}^{\text{sat}} = & h_1[S(n_1^2\beta_1^2) - \frac{1}{3}] + 2h_2S(n_1n_2\beta_1\beta_2) + h_3[S(n_1^2n_2^2) - \frac{1}{3}] \\ & + h_4[S(n_1^4\beta_1^2 + \frac{2}{3}n_1^2n_2^2) - \frac{1}{3}] + 2h_5S(n_1^2n_2n_3\beta_2\beta_3); \end{aligned} \quad (1.5.9.19)$$

$\langle 100 \rangle$  type:

$$\begin{aligned} \lambda_{\beta}^{\text{sat}} = & h_1[S(n_1^2\beta_1^2) - \frac{1}{3}] + 2h_2S(n_1n_2\beta_1\beta_2) + h_3S(n_1^2n_2^2) \\ & + h_4[S(n_1^4\beta_1^2 + \frac{2}{3}n_1^2n_2^2) - \frac{1}{3}] + 2h_5S(n_1^2n_2n_3\beta_2\beta_3). \end{aligned} \quad (1.5.9.20)$$

Both types of crystals with a cubic prototype are described by a two-constant equation if the terms of fourth power are neglected. This equation was obtained by Akulov (1928) in the form

$$\begin{aligned} \lambda_{\beta}^{\text{sat}} = & \frac{2}{3}\lambda_{100}(n_1^2\beta_1^2 + n_2^2\beta_2^2 + n_3^2\beta_3^2 - \frac{1}{3}) \\ & + 3\lambda_{111}(n_1n_2\beta_1\beta_2 + n_2n_3\beta_2\beta_3 + n_3n_1\beta_3\beta_1), \end{aligned} \quad (1.5.9.21)$$

where the constants  $\lambda_{100}$  and  $\lambda_{111}$  correspond to the magnetostrictive deformation of a 'cubic' ferromagnet along the direction of the magnetic field that is applied along the directions  $\langle 100 \rangle$  and  $\langle 111 \rangle$ , respectively. Let us denote by  $Q_1$  and  $Q_2$  the following equal coefficients in the equation for the magnetoelastic energy (1.5.9.3):

$$Q_1 = Q_{xxxx} = Q_{yyyy} = Q_{zzzz}; \quad Q_2 = Q_{xyxy} = Q_{yzyz} = Q_{zxzx}. \quad (1.5.9.22)$$

According to (1.5.9.9), the coefficients  $\lambda_{100}$  and  $\lambda_{111}$  may be written as the following fractions of  $Q_i$  and the elastic stiffnesses  $c_{\alpha\beta}$ :

$$\lambda_{100} = \frac{Q_1}{c_{12} - c_{11}}, \quad \lambda_{111} = -\frac{1}{3} \frac{Q_2}{c_{44}}. \quad (1.5.9.23)$$

If the magnetic field transforms the crystal from the demagnetized to the saturated state and if the linear dimension of the sample along the magnetic field increases, then its dimension perpendicular to the field will decrease (see Fig. 1.5.9.1). It follows from relation (1.5.9.21) that the magnetostriction perpendicular to the magnetic field is

$$\lambda_{100}^{\perp} = -\frac{1}{2}\lambda_{100} \quad \text{and} \quad \lambda_{111}^{\perp} = -\frac{1}{2}\lambda_{111}. \quad (1.5.9.24)$$

Some data for magnetostriction of ferromagnets with prototype symmetry  $m\bar{3}m1'$  are presented in Table 1.5.9.2.

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Table 1.5.10.1. Conversion of non-rationalized (except for  $\alpha$ ) Gaussian units to SI units

Symbol	Quantity	Gaussian unit and its SI equivalent
<b>B</b>	Magnetic induction	1 gauss (G) = $10^{-4}$ tesla (T)
<b>H</b>	Magnetic field	1 oersted (Oe) = $10^3/(4\pi)$ A m $^{-1}$
<b>M</b>	Magnetization (= magnetic moment per unit volume)	1 emu cm $^{-3}$ = $10^3$ A m $^{-1}$
$\alpha$	Linear magnetoelectric tensor (rationalized units)	1 (dimensionless units) = $4\pi \times 10^{-8}/3$ s m $^{-1}$
$\Lambda$	Piezomagnetic tensor	1 Oe $^{-1}$ = $4\pi \times 10^{-3}$ m A $^{-1}$ = $4\pi \times 10^{-3}$ T Pa $^{-1}$
$\chi$	Magnetic volume susceptibility	1 (dimensionless units) = $4\pi$ (dimensionless units)
$\chi_g$	Magnetic mass susceptibility	1 cm $^3$ g $^{-1}$ = $4\pi \times 10^{-3}$ m $^3$ kg $^{-1}$
$\chi_{mol}$	Magnetic molar susceptibility	1 cm $^3$ mol $^{-1}$ = $4\pi \times 10^{-6}$ m $^3$ mol $^{-1}$

In a uniaxial crystal, the magnetostriction in the magnetic field arises mainly as a result of the rotation of the magnetization vector from the direction of the easy axis to the direction of the applied field. The magnetostriction in the magnetic field of an easy-axis hexagonal ferromagnet can be obtained from the relation for the spontaneous magnetostriction (1.5.9.14). In the demagnetized state, such a ferromagnet possesses only two types of antiparallel domains, in which the magnetization is aligned parallel or antiparallel to the hexagonal axis ( $n_z = \pm 1$ ,  $n_x = n_y = 0$ ).

Thus the magnetostriction of the demagnetized state is described by

$$\lambda_\beta^{\text{dem}} = h_0 + (h_1 + h_6)\beta_3^2. \quad (1.5.9.25)$$

The saturation magnetostriction can be calculated for different directions of the applied magnetic field using the equations (1.5.9.14), (1.5.9.15) and (1.5.9.25). If the magnetic field is applied along the  $x$  axis ( $n_x = 1$ ,  $n_y = n_z = 0$ ), the saturation magnetostrictions for three directions of the vector  $\beta$ :  $\lambda_\beta^{\text{sat}} = \lambda_A, \lambda_B, \lambda_C$  are

$$\begin{aligned} \beta \parallel Ox \quad \lambda_A &= h_2, \\ \beta \parallel Oy \quad \lambda_B &= h_3, \\ \beta \parallel Oz \quad \lambda_C &= -h_1. \end{aligned} \quad (1.5.9.26)$$

If the magnetic field is applied at an angle of  $45^\circ$  to the hexagonal axis along the [101] direction, the saturation magnetostriction along the magnetic field is described by

$$\lambda_D = \lambda_{101}^{\text{sat}} = \frac{1}{4}(h_2 - h_1 + 2h_5). \quad (1.5.9.27)$$

Using the constants  $\lambda_A, \lambda_B, \lambda_C$  and  $\lambda_D$  introduced above, the general relation for the magnetostriction caused by magnetization to saturation can be presented in the form

$$\begin{aligned} \lambda_\beta^{\text{sat}} &= \lambda_A[(n_1\beta_1 + n_2\beta_2)^2 - (n_1\beta_1 + n_2\beta_2)n_3\beta_3] \\ &+ \lambda_B[(1 - n_3^2)(1 - \beta_3^2) - (n_1\beta_1 + n_2\beta_2)^2] \\ &+ \lambda_C[(1 - n_3^2)\beta_3^2 - (n_1\beta_1 + n_2\beta_2)n_3\beta_3] \\ &+ 4\lambda_D(n_1\beta_1 + n_2\beta_2)n_3\beta_3. \end{aligned} \quad (1.5.9.28)$$

A typical hexagonal ferromagnet is cobalt. The magnetostriction constants introduced above have the following values for Co at room temperature:

$$\begin{aligned} \lambda_A &= -45 \times 10^{-6}, & \lambda_C &= +110 \times 10^{-6}, \\ \lambda_B &= -95 \times 10^{-6}, & \lambda_D &= -100 \times 10^{-6}. \end{aligned}$$

A more sophisticated treatment of the symmetry of the magnetostriction constants is given in the monograph of Birss (1964) and in Zalesky (1981).

## 1.5.9.3. The difference between the magnetic anisotropies at zero strain and zero stress

The spontaneous magnetostriction makes a contribution to the magnetic anisotropy (especially in crystals with a cubic prototype). Therefore, to find the full expression for the anisotropy energy, one has to sum up the magnetic  $U_a^0$  [see (1.5.9.5)], the magnetoelastic  $U_{me}$  [see (1.5.9.3)] and the elastic  $U_{el}$  [see (1.5.9.6)] energies. At zero strain ( $S_{ij}^* = 0$ ), only  $U_a^0 \neq 0$ . At zero stress

$$\begin{aligned} U_a^0 + U_{me} + U_{el} &= U_a^0 + V_{ij}^0 c_{ij}^* + \frac{1}{2} c_{ijkl} S_{ij}^* S_{kl}^* \\ &= U_a^0 + \frac{1}{2} V_{ij}^0 S_{ij}^*. \end{aligned} \quad (1.5.9.29)$$

Here we used the modified equation (1.5.9.7):

$$\frac{1}{2} c_{ijkl} S_{ij}^* S_{kl}^* = -\frac{1}{2} V_{ij}^0 S_{ij}^*. \quad (1.5.9.30)$$

Substituting the values for the spontaneous magnetostriction, the final equation for the anisotropy energy measured at atmospheric pressure may be written as

$$\begin{aligned} U_a &= U_a^0 + \frac{1}{2} V_{ij}^0 c_{ij}^* \\ &= (K_{ij}^0 + K'_{ij})n_i n_j + (K_{ijkl}^0 + K'_{ijkl})n_i n_j n_k n_l \\ &\quad + (K_{ijklmn}^0 + K'_{ijklmn})n_i n_j n_k n_l n_m n_n + \dots \end{aligned} \quad (1.5.9.31)$$

As an example, for the ferromagnets with a cubic prototype this equation may be written as

$$U_a = (K_1^0 + K_1')S(n_1^2 n_2^2) + (K_2^0 + K_2')n_1^2 n_2^2 n_3^2. \quad (1.5.9.32)$$

The coefficients  $K_1'$  and  $K_2'$  may be expressed in terms of the saturation magnetostriction constants  $h_0, \dots, h_5$  [see (1.5.9.12)] and the elastic stiffnesses  $c_{\alpha\beta}$ :

$$\begin{aligned} K_1' &= c_{11}[h_0(2h_4 - 3h_3) + h_1(h_1 - h_3 + 3h_4) - h_4(h_3 - 2h_4)] \\ &\quad + c_{12}[2h_0(2h_4 - 3h_3) - (h_1 + h_4)(h_1 + 2h_3)] - \frac{1}{2} c_{44} h_2^2, \end{aligned} \quad (1.5.9.33)$$

$$\begin{aligned} K_2' &= -c_{11}[3h_4(h_1 + h_3) + (h_4 - h_3)(4h_4 - 3h_3)] \\ &\quad + c_{12}[3h_4(h_1 + h_3) + h_3(5h_4 - 6h_3)] \\ &\quad - \frac{1}{2} c_{44}(6h_2 + h_5)h_5. \end{aligned} \quad (1.5.9.34)$$

For cubic crystals,  $K_i^0$  and  $K_i'$  are of the same magnitude. As an example, for Ni one has  $K_1^0 = 80\,000$  erg cm $^{-3}$  =  $8000$  J m $^{-3}$  and  $K_1' = -139\,000$  erg cm $^{-3}$  =  $-13\,900$  J m $^{-3}$ .

## 1.5.10. Connection between Gaussian and SI units

Numerical values of magnetic quantities are given in the tables and figures in this chapter in Gaussian units together with information on how the corresponding values in SI units are obtained. As a summary, Table 1.5.10.1 gives for each such quantity the corresponding Gaussian unit and its value expressed in SI units. More details on the transformation between Gaussian and SI units are given *e.g.* in the Appendix of Jackson (1999).