

## 1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

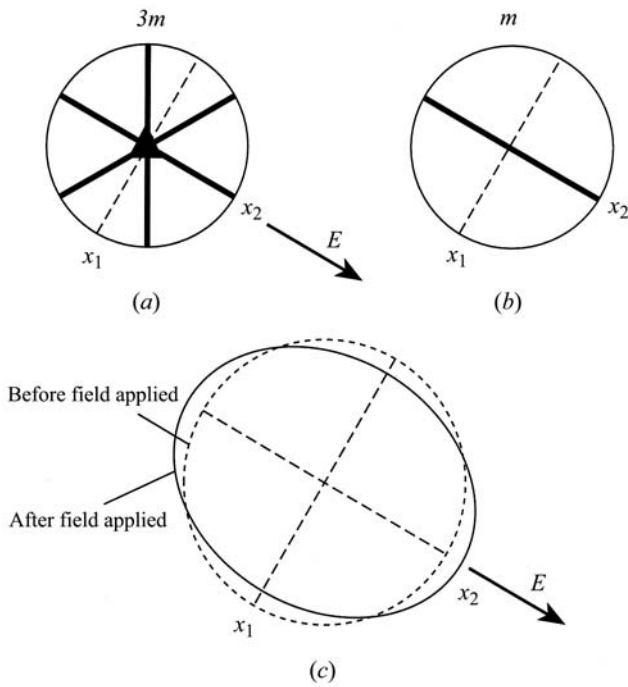


Fig. 1.6.6.1. (a) Symmetry elements of point group  $3m$ . (b) Symmetry elements after field applied along  $x_2$ . (c) Effect on circular section of uniaxial indicatrix.

If, on the other hand, the electric field  $E_2^0$  is applied along  $x_2$ , i.e. within the mirror plane, one finds

$$\begin{aligned}\Delta\eta_1 &= -r_{22}E_2^0 \\ \Delta\eta_2 &= +r_{22}E_2^0 \\ \Delta\eta_4 &= +r_{51}E_2^0 \\ \Delta\eta_3 &= \Delta\eta_5 = \Delta\eta_6 = 0.\end{aligned}\quad (1.6.6.9)$$

Diagonalization of the matrix

$$\begin{pmatrix} \Delta\eta_1 & 0 & 0 \\ 0 & \Delta\eta_2 & \Delta\eta_4 \\ 0 & \Delta\eta_4 & \Delta\eta_3 \end{pmatrix}\quad (1.6.6.10)$$

containing these terms gives three eigenvalue solutions for the changes in dielectric impermeabilities:

$$\begin{aligned}(1) & \quad -r_{22}E_2^0 \\ (2) & \quad \frac{r_{22} + (r_{22}^2 + 4r_{51}^2)^{1/2}}{2} E_2^0 \\ (3) & \quad \frac{r_{22} - (r_{22}^2 + 4r_{51}^2)^{1/2}}{2} E_2^0.\end{aligned}\quad (1.6.6.11)$$

On calculating the eigenfunctions, it is found that solution (1) lies along  $x_1$ , thus representing a change in the value of the indicatrix axis in this direction. Solutions (2) and (3) give the other two axes of the indicatrix: these are different in length, but mutually perpendicular, and lie in the  $x_2x_3$  plane. Thus a biaxial indicatrix is formed with one refractive index fixed along  $x_1$  and the other two in the plane perpendicular. The effect of having the electric field imposed within the mirror plane is thus to remove the threefold axis in point group  $3m$  and to form the point subgroup  $m$  (Fig. 1.6.6.1).

Relationship between linear electro-optic coefficients  $r_{ijk}$  and the susceptibility tensor  $\chi_{ijk}^{(2)}$ . It is instructive to repeat the above calculation using the normal susceptibility tensor and equation (1.6.3.14). Consider, again, a static electric field along  $x_3$  and light propagating along  $x_1$ . As before, the only coefficients that need to

be considered with the static field along  $x_3$  are  $\chi_{113} = \chi_{223}$  and  $\chi_{333}$ . Equation (1.6.3.14) can then be written as

$$\begin{pmatrix} \varepsilon_1 + \varepsilon_o\chi_{13}E_3^0 + n^2 & 0 & 0 \\ 0 & \varepsilon_1 + \varepsilon_o\chi_{13}E_3^0 & 0 \\ 0 & 0 & \varepsilon_3 + \varepsilon_o\chi_{33}E_3^0 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix},\quad (1.6.6.12)$$

where for simplicity the Voigt notation has been used. The first line of the matrix equation gives

$$(\varepsilon_1 + \varepsilon_o\chi_{13}E_3^0 + n^2)E_1 = n^2E_1.\quad (1.6.6.13)$$

Since only a transverse electric field is relevant for an optical wave (plasma waves are not considered here), it can be assumed that the longitudinal field  $E_1 = 0$ . The remaining two equations can be solved by forming the determinantal equation

$$\begin{vmatrix} \varepsilon_1 + \varepsilon_o\chi_{13}E_3^0 - n^2 & 0 \\ 0 & \varepsilon_3 + \varepsilon_o\chi_{33}E_3^0 - n^2 \end{vmatrix} = 0,\quad (1.6.6.14)$$

which leads to the results

$$n_1^2 = \varepsilon_1 + \varepsilon_o\chi_{13}E_3^0 \quad \text{and} \quad n_2^2 = \varepsilon_3 + \varepsilon_o\chi_{33}E_3^0.\quad (1.6.6.15)$$

Thus

$$n_1^2 = n_o^2 + \varepsilon_o\chi_{13}E_3^0 \quad \text{and} \quad n_2^2 = n_e^2 + \varepsilon_o\chi_{33}E_3^0,\quad (1.6.6.16)$$

and so

$$(n_1 - n_o)(n_1 + n_o) = \varepsilon_o\chi_{13}E_3^0 \quad \text{and} \quad (n_2 - n_e)(n_2 + n_e) = \varepsilon_o\chi_{33}E_3^0,\quad (1.6.6.17)$$

and since  $n_1 \simeq n_o$  and  $n_2 \simeq n_e$ ,

$$n_1 - n_o = \frac{\varepsilon_o\chi_{13}E_3^0}{2n_o} \quad \text{and} \quad n_2 - n_e = \frac{\varepsilon_o\chi_{33}E_3^0}{2n_e}.\quad (1.6.6.18)$$

Subtracting these two results, the induced birefringence is found:

$$(n_e - n_o) - \frac{1}{2} \left( \frac{\varepsilon_o\chi_{33}}{n_e} - \frac{\varepsilon_o\chi_{13}}{n_o} \right) E_3^0.\quad (1.6.6.19)$$

Comparing with the equation (1.6.6.8) calculated for the linear electro-optic coefficients,

$$(n_e - n_o) - \frac{1}{2}(n_e^3r_{33} - n_o^3r_{13})E_3^0,\quad (1.6.6.20)$$

one finds the following relationships between the linear electro-optic coefficients and the susceptibilities  $\chi^{(2)}$ :

$$r_{13} = \frac{\varepsilon_o\chi_{13}}{n_o^4} \quad \text{and} \quad r_{33} = \frac{\varepsilon_o\chi_{33}}{n_e^4}.\quad (1.6.6.21)$$

## 1.6.7. The linear photoelastic effect

### 1.6.7.1. Introduction

The linear photoelastic (or piezo-optic) effect (Narasimhamurthy, 1981) is given by  $P_i^\omega = \varepsilon_o\chi_{ijk}E_j^\omega S_{kl}^0$ , and, like the electro-optic effect, it can be discussed in terms of the change in dielectric impermeability caused by a static (or low-frequency) field, in this case a stress, applied to the crystal. This can be written in the form

$$\Delta\eta_{ij} = \pi_{ijkl}T_{kl}^0.\quad (1.6.7.1)$$

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The coefficients  $\pi_{ijkl}$  form a fourth-rank tensor known as the *linear piezo-optic tensor*. Typically, the piezo-optic coefficients are of the order of  $10^{-12} \text{ m}^2 \text{ N}^{-1}$ . It is, however, more usual to express the effect as an *elasto-optic effect* by making use of the relationship between stress and strain (see Section 1.3.3.2), thus

$$T_{kl} = c_{klmn} S_{mn}, \quad (1.6.7.2)$$

where the  $c_{klmn}$  are the elastic stiffness coefficients. Therefore equation (1.6.7.2) can be rewritten in the form

$$\Delta\eta_{ij} = \pi_{ijkl} c_{klmn} S_{mn} = p_{ijmn} S_{mn} \quad (1.6.7.3)$$

or, in contracted notation,

$$\Delta\eta_i = p_{ij} S_j, \quad (1.6.7.4)$$

where, for convenience, the superscript 0 has been dropped, the elastic strain being considered as essentially static or of low frequency compared with the natural mechanical resonances of the crystal. The  $p_{ijmn}$  are coefficients that form the *linear elasto-optic* (or *strain-optic*) *tensor* (Table 1.6.7.1). Note that these coefficients are dimensionless, and typically of order  $10^{-1}$ , showing that the change to the optical indicatrix is roughly one-tenth of the strain.

The elasto-optic effect can arise in several ways. The most obvious way is through application of an external stress, applied to the surfaces of the crystal. However, strains, and hence changes to the refractive indices, can arise in a crystal through other ways that are less obvious. Thus, it is a common finding that crystals can be twinned, and thus the boundary between twin domains, which corresponds to a mismatch between the crystal structures either side of the domain boundary, will exhibit a strain. Such a crystal, when viewed between crossed polars under a microscope will produce birefringence colours that will highlight the contrast between the domains. This is known as *strain birefringence*. Similarly, when a crystal undergoes a phase transition involving a change in crystal system, a so-called *ferroelastic transition*, there will be a change in strain owing to the difference in unit-cell shapes. Hence there will be a corresponding change in the optical indicatrix. Often the phase transition is one going from a high-temperature optically isotropic section to a low-temperature optically anisotropic section. In this case, the high-temperature section has no internal strain, but the low-temperature phase acquires a strain, which is often called the *spontaneous strain* (by analogy with the term spontaneous polarization in ferroelectrics).

### 1.6.7.2. Spontaneous strain in $\text{BaTiO}_3$

As an example of the calculation of the relationship between spontaneous strain and linear birefringence, consider the high-temperature phase transition of the well known perovskite  $\text{BaTiO}_3$ . This substance undergoes a transition at around 403 K on cooling from its high-temperature  $Pm\bar{3}m$  phase to the room-temperature  $P4mm$  phase. The  $P4mm$  phase is both ferroelectric and ferroelastic. In this tetragonal phase, there is a small distortion of the unit cell along  $[001]$  and a contraction along  $\langle 100 \rangle$  compared with the unit cell of the high-temperature cubic phase, and so the room-temperature phase can be expected to have a uniaxial optical indicatrix.

The elasto-optic tensor for the  $m\bar{3}m$  phase is (Table 1.6.7.1)

$$\begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}. \quad (1.6.7.5)$$

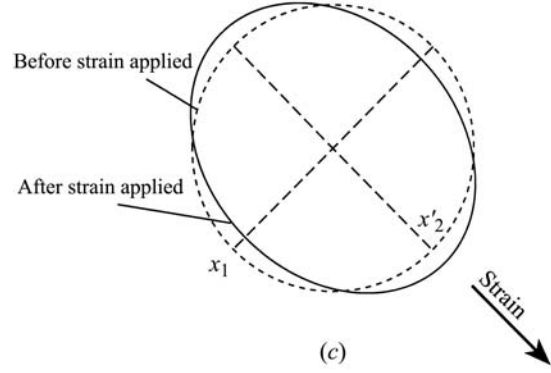
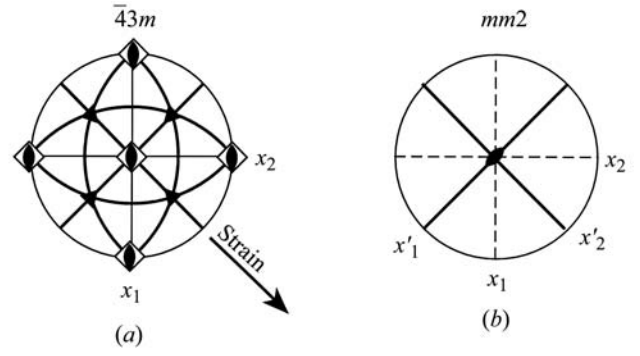


Fig. 1.6.7.1. (a) Symmetry elements of point group  $\bar{4}3m$ . (b) Symmetry elements after strain applied along  $[110]$ . (c) Effect on spherical indicatrix.

Consider the low-temperature tetragonal phase to arise as a small distortion of this cubic phase, with a spontaneous strain  $S_3^s$  given by the lattice parameters of the tetragonal phase:

$$S_3^s = [(c - a)/a]. \quad (1.6.7.6)$$

Therefore, the equations (1.6.7.4) for the dielectric impermeability in terms of the spontaneous strain component are given in matrix form as

$$\begin{pmatrix} \Delta\eta_1 \\ \Delta\eta_2 \\ \Delta\eta_3 \\ \Delta\eta_4 \\ \Delta\eta_5 \\ \Delta\eta_6 \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ S_3^s \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} p_{12} S_3^s \\ p_{12} S_3^s \\ p_{11} S_3^s \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (1.6.7.7)$$

so that

$$\begin{aligned} \Delta\eta_1 &= \Delta\eta_2 = p_{12} S_3^s \\ \Delta\eta_3 &= p_{11} S_3^s \\ \Delta\eta_4 &= \Delta\eta_5 = \Delta\eta_6 = 0. \end{aligned} \quad (1.6.7.8)$$

By analogy with equations (1.6.6.5) and (1.6.6.6), the induced changes in refractive index are then

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Table 1.6.7.1. Symmetry constraints on the linear elasto-optic (strain-optic) tensor  $p_{ij}$  (contracted notation) (see Section 1.1.4.10.6)

| Triclinic  | Orthorhombic   | Tetragonal  | Trigonal  |
|--|--|---|---|
| Point group 1  | Point groups 222, $mm2$ , $mmm$  | Point groups 4, $\bar{4}$ , $4/m$   | Point groups 3, $\bar{3}$   |
| $\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} \end{pmatrix}$ | $\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & 0 & 0 \\ p_{31} & p_{32} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{pmatrix}$   | $\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{12} & p_{11} & p_{13} & 0 & 0 & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & -p_{45} & p_{44} & 0 \\ p_{61} & -p_{61} & 0 & 0 & 0 & p_{66} \end{pmatrix}$ | $\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{12} & p_{11} & p_{13} & -p_{14} & -p_{15} & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ p_{41} & -p_{41} & 0 & p_{44} & p_{45} & -p_{51} \\ p_{51} & -p_{51} & 0 & -p_{45} & p_{44} & p_{41} \\ -p_{16} & p_{16} & 0 & -p_{15} & p_{14} & \frac{1}{2}(p_{11} - p_{12}) \end{pmatrix}$ |
| Monoclinic   | Point groups 2, $m$ , $2/m$ ( $2 \parallel x_2$ )  | Point groups $4mm$ , $\bar{4}2m$ , $422$ , $4/mmm$  | Point groups $3m$ , $\bar{3}m$ , $32$   |
| $\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & p_{15} & 0 \\ p_{21} & p_{22} & p_{23} & 0 & p_{25} & 0 \\ p_{31} & p_{32} & p_{33} & 0 & p_{35} & 0 \\ 0 & 0 & 0 & p_{44} & 0 & p_{46} \\ p_{51} & p_{52} & p_{53} & 0 & p_{55} & 0 \\ 0 & 0 & 0 & p_{64} & 0 & p_{66} \end{pmatrix}$   | $\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{21} & p_{22} & p_{23} & 0 & 0 & p_{26} \\ p_{31} & p_{32} & p_{33} & 0 & 0 & p_{36} \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & 0 & p_{54} & p_{55} \\ p_{61} & p_{62} & p_{63} & 0 & 0 & p_{66} \end{pmatrix}$ | $\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{pmatrix}$                                  | $\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 & 0 \\ p_{12} & p_{11} & p_{13} & -p_{14} & 0 & 0 \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ p_{41} & -p_{41} & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & p_{41} \\ 0 & 0 & 0 & 0 & p_{14} & \frac{1}{2}(p_{11} - p_{12}) \end{pmatrix}$  |

| Hexagonal   | Cubic  | Isotropic  |
|---|--|--|
| Point groups 6, $\bar{6}$ , $6/m$   | Point groups $m\bar{3}$ , $23$   |  |
| $\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{12} & p_{11} & p_{13} & 0 & 0 & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & -p_{45} & p_{44} & 0 \\ -p_{16} & p_{16} & 0 & 0 & 0 & \frac{1}{2}(p_{11} - p_{12}) \end{pmatrix}$ | $\begin{pmatrix} p_{11} & p_{12} & p_{21} & 0 & 0 & 0 \\ p_{21} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{21} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}$ | $\begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(p_{11} - p_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11} - p_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11} - p_{12}) \end{pmatrix}$ |
| Point groups $6mm$ , $\bar{6}m2$ , $622$ , $6/mmm$  | Point groups $\bar{4}3m$ , $432$ , $m\bar{3}m$   |  |
| $\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11} - p_{12}) \end{pmatrix}$                                  | $\begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}$ |  |

$$\Delta n_1 = \Delta n_2 = -\frac{n_{\text{cub}}^3}{2} p_{12} S_3^s$$

$$\Delta n_3 = -\frac{n_{\text{cub}}^3}{2} p_{11} S_3^s, \quad (1.6.7.9)$$

where  $n_{\text{cub}}$  is the refractive index of the cubic phase. Thus the birefringence in the tetragonal phase as seen by light travelling along  $x_1$  is given by

$$\Delta n_3 - \Delta n_2 = -\frac{n_{\text{cub}}^3}{2} (p_{11} - p_{12}) S_3^s. \quad (1.6.7.10)$$

Thus a direct connection is made between the birefringence of the tetragonal phase of  $\text{BaTiO}_3$  and its lattice parameters *via* the spontaneous strain. As in the case of the linear electro-optic effect, the calculation can be repeated using equation (1.6.3.14) with the susceptibilities  $\chi_{11}$  and  $\chi_{12}$  to yield the relationship

$$p_{11} = \frac{c_{1111} \epsilon_o \chi_{11}}{n_o^4}; \quad p_{12} = \frac{c_{1122} \epsilon_o \chi_{12}}{n_o^4}. \quad (1.6.7.11)$$

### 1.6.7.3. The acousto-optic effect

The acousto-optic effect (Sapriel, 1976) is really a variant of the elasto-optic effect, in that the strain field is created by the passage of a sound wave through the crystal. If this wave has frequency  $\omega_1$ , the resulting polarization in the presence of a light

wave of frequency  $\omega_2$  is given by  $P_i^o = \chi_{ijkl} E_j^{\omega_2} S_{kl}^{\omega_1}$ , where  $\omega = \omega_1 \pm \omega_2$ . However, since the sound-wave frequency is very small compared with that of the light, to all intents and purposes the change in frequency of the light field can be ignored. The effect then of the sound wave is to produce within an acousto-optic crystal a spatially modulated change in refractive index: a beam of light can then be diffracted by this spatial modulation, the resulting optical diffraction pattern thus changing with the changing sound signal. Acousto-optic materials therefore can be used as transducers for converting sound signals into optical signals for transmission down optical fibres in communications systems. Consider, for instance, a sound wave propagating along the [110] direction in gallium arsenide ( $\text{GaAs}$ ), which crystallizes in point group  $43m$ . Suppose that this sound wave is longitudinally polarized. With respect to the cube axes, this corresponds to an oscillatory shear strain  $S_{12} \sin(\omega t - k\xi)$ , where  $\xi$  is a distance along the [110] direction (Fig. 1.6.7.1). Then one can write

$$\Delta \eta_{ij} = p_{ij12} S_{12} \sin(\omega t - k\xi) \quad (1.6.7.12)$$

or in contracted notation

$$\Delta \eta_i = p_{i6} S_6 \sin(\omega t - k\xi). \quad (1.6.7.13)$$

From Table 1.6.7.1, it is seen that the change in dielectric impermeability tensor is

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$$\Delta\eta_6 = p_{66}S_6 \sin(\omega t - k\xi) = p_{44}S_6 \sin(\omega t - k\xi) \quad (1.6.7.14)$$

since all other components are zero. This means that the original spherical indicatrix of the cubic crystal has been distorted to form a biaxial indicatrix whose axes oscillate in length according to

$$\begin{aligned} n_1 &= n_{\text{cub}} + \frac{n_{\text{cub}}^3}{2} p_{44} S_6 \sin(\omega t - k\xi) \\ n_2 &= n_{\text{cub}} - \frac{n_{\text{cub}}^3}{2} p_{44} S_6 \sin(\omega t - k\xi) \\ n_3 &= n_{\text{cub}}, \end{aligned} \quad (1.6.7.15)$$

thus forming an optical grating of spatial periodicity given by the  $k\xi$  term. In gallium arsenide, at a wavelength of light equal to  $1.15 \mu\text{m}$ ,  $p_{11} = -0.165$ ,  $p_{12} = -0.140$  and  $p_{44} = -0.072$ . It is convenient to define a figure of merit for acousto-optic materials (Yariv & Yeh, 1983) given by

$$M = \frac{n^6 p^2}{d v^3}, \quad (1.6.7.16)$$

where  $v$  is the velocity of the sound wave and  $d$  is the density of the solid. For gallium arsenide,  $d = 5340 \text{ kg m}^{-3}$ , and for a sound wave propagating as above  $v = 5.15 \text{ m s}^{-1}$ . At the wavelength  $\lambda = 1.15 \mu\text{m}$ ,  $n = 3.37$ , and so it is found that  $M = 104$ . In practice, figures of merits can range from less than 0.001 up to as high as 4400 in the case of Te, and so the value for gallium arsenide makes it potentially useful as an acousto-optic material for infrared signals.

### 1.6.8. Glossary

|                               |   |
|-------------------------------|---|
| $\alpha, \beta, \gamma$       | refractive indices of biaxial indicatrix, $\alpha < \beta < \gamma$           |
| $\hat{\alpha}$                | polarizability operator   |
| $B_i$                         | $i$ th component of magnetic induction  |
| $c$                           | velocity of light   |
| $c_{klmn}$                    | $klmn$ th component of elastic stiffness tensor                               |
| $\chi_{ijk\dots}$             | $ijk\dots$ th component of generalized susceptibility                         |
| $d$                           | density   |
| $D_i$                         | $i$ th component of dielectric displacement                                   |
| $\Delta$                      | phase difference of light   |
| $\hat{e}_{ijm}$               | unit antisymmetric pseudotensor of rank 3                                     |
| $E_i$                         | $i$ th component of electric field  |
| $g_{ij}, G_{ij}$              | $ij$ th component of gyration tensor  |
| $\mathbf{G}$                  | gyration vector   |
| $\gamma_{ij\ell}$             | third-rank optical gyration susceptibility                                    |
| $\mathbf{H}$                  | magnetic field intensity  |
| $\eta_{ij}$                   | $ij$ th component of dielectric impermeability tensor                         |
| $\epsilon_o$                  | permittivity of free space  |
| $\epsilon_{ij}$               | $ij$ th component of dielectric tensor  |
| $\kappa$                      | ellipticity of wave   |
| $\mathbf{k}$                  | wavevector of light propagating in crystal<br>( $ k  = 2\pi/\lambda$ )        |
| $\lambda$                     | wavelength of light   |
| $\mu_o$                       | vacuum magnetic permeability  |
| $n$                           | refractive index of light   |
| $n_\alpha, n_\beta, n_\gamma$ | refractive indices for biaxial indicatrix,<br>$n_\alpha < n_\beta < n_\gamma$ |
| $n_o$                         | ordinary refractive index   |
| $n_e$                         | extraordinary refractive index  |
| $\Psi_i$                      | wavefunction of state $i$   |
| $P_i$                         | $i$ th component of electric polarization                                     |
| $p_{ijkl}$                    | $ijkl$ th component of elasto-optic (strain-optic) tensor                     |
| $\hat{p}$                     | electric dipole operator  |

|              |   |
|--------------|---|
| $\rho$       | optical rotatory power                                |
| $\pi_{ijkl}$ | $ijkl$ th component of linear piezo-optic tensor      |
| $r_{ijk}$    | $ijk$ th component of linear electro-optic tensor     |
| $\hat{s}$    | unit vector in the direction of $s$ , the wave normal |
| $S_{ij}$     | $ij$ th component of strain tensor                    |
| $T_{ij}$     | $ij$ th component of stress tensor                    |
| $v$          | velocity of sound                                     |
| $V$          | half the angle between optic axes                     |
| $\omega$     | cyclic frequency                                      |
| $x_i$        | direction of $i$ th Cartesian axis, $i = 1, 2, 3$     |

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