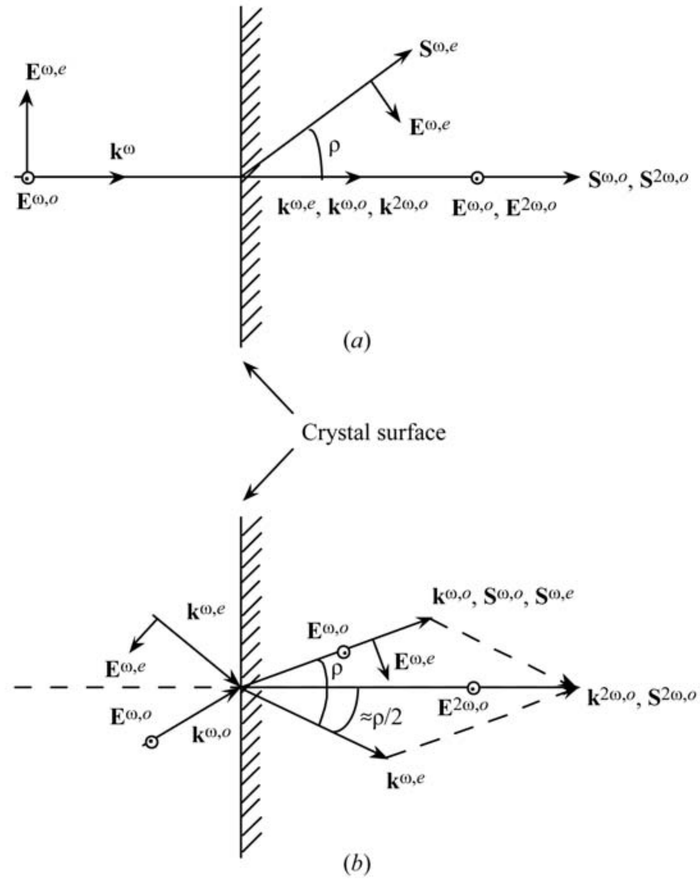

Fig. 1.7.3.11. Comparison between (*a*) collinear and (*b*) special non-collinear phase matching for (*oeo*) type-II SHG. $\mathbf{k}^{\omega,o}$, $\mathbf{k}^{\omega,e}$ and $\mathbf{k}^{2\omega,o}$ are the wavevectors, $\mathbf{S}^{\omega,o}$, $\mathbf{S}^{\omega,e}$ and $\mathbf{S}^{2\omega,o}$ are the Poynting vectors of the fundamental and harmonic waves, and $\mathbf{E}^{\omega,o}$, $\mathbf{E}^{\omega,e}$ and $\mathbf{E}^{2\omega,o}$ are the electric field vectors; ρ is the walk-off angle in the collinear case and the angle between $\mathbf{k}^{\omega,o}$ and $\mathbf{k}^{\omega,e}$ inside the crystal for the non-collinear interaction.

because the saturation phenomenon of type-II CPM is avoided.

(iv) *Effect of temporal walk-off*.

Even if the SHG is phase matched, the fundamental and harmonic group velocities, $v_g(\omega) = \partial\omega/\partial k$, are generally mismatched. This has no effect with continuous wave (c.w.) lasers. For pulsed beams, the temporal separation of the different beams during the propagation can lead to a decrease of the temporal overlap of the pulses. Indeed, this walk-off in the time domain affects the conversion efficiency when the pulse separations are close to the pulse durations. Then after a certain distance, L_τ, the pulses are completely separated, which entails a saturation of the conversion efficiency, for both types I and II (Tomov *et al.*, 1982). Three group velocities must be considered for type II. Type I is simpler, because the two fundamental waves have the same velocity, so $L_\tau = \tau/[v_g^{-1}(\omega) - v_g^{-1}(2\omega)]$, which defines the optimum crystal length, where τ is the pulse duration. For type-I SHG of 532 nm in KH_2PO_4 (KDP), $v_g(266\text{ nm}) = 1.84 \times 10^8\text{ m s}^{-1}$ and $v_g(532\text{ nm}) = 1.94 \times 10^8\text{ m s}^{-1}$, so $L_\tau = 3.5$ mm for 1 ps. For the usual nonlinear crystals, the temporal walk-off must be taken into account for pico- and femtosecond pulses.

1.7.3.3.2.2. *Non-resonant SHG with undepleted pump and transverse and longitudinal Gaussian beams*

We now consider the general situation where the crystal length can be larger than the Rayleigh length.

The Gaussian electric field amplitudes of the two eigen electric field vectors inside the nonlinear crystal are given by

$$E^{\pm}(X,Y,Z) = E_o^{\pm}\frac{w_o}{w(Z)}\exp\left[-\frac{(X+\rho^+Z)^2+(Y+\rho^-Z)^2}{w^2(Z)}\right] \times \exp\left(i\left\{k^{\pm}Z - \arctan(Z/z_R) + \frac{k^{\pm}\left[(X+\rho^+Z)^2+(Y+\rho^-Z)^2\right]}{2Z\left[1+(z_R^2/Z^2)\right]}\right\}\right) \quad (1.7.3.55)$$

with $\rho^- = 0$ for E^+ and $\rho^+ = 0$ for E^-.

(X, Y, Z) is the wave frame defined in Fig. 1.7.3.1. $E_o^{\pm}$ is the scalar complex amplitude at $(X, Y, Z) = (0, 0, 0)$ in the vibration planes $\Pi^{\pm}$.

We consider the refracted waves E^+ and E^- to have the same longitudinal profile inside the crystal. Then the $(1/e^2)$ beam radius is given by $w(Z)^2 = w_o^2[1 + (Z^2/z_R^2)]$, where w_o is the minimum beam radius located at $Z = 0$ and $z_R = kw_o^2/2$, with $k = (k^+ + k^-)/2$; z_R is the Rayleigh length, the length over which the beam radius remains essentially collimated; $k^{\pm}$ are the wavevectors at the wavelength λ in the direction of propagation Z. The far-field half divergence angle is $\Delta\alpha = 2/kw_o$.

The coordinate systems of (1.7.3.22) are identical to those of the parallel-beam limit defined in (iii).

In these conditions and by assuming the undepleted pump approximation, the integration of (1.7.3.22) over (X, Y, Z) leads to the following expression of the power conversion efficiency (Zondy, 1991):

$$\eta_{\rm SHG}(L) = \frac{P^{2\omega}(L)}{P^{\omega}(0)} = CLP^{\omega}(0)\frac{h(L, w_o, \rho, f, \Delta k)}{\cos^2\rho_{2\omega}}$$

with

$$C = 5.95\times10^{-2}\,\frac{2N-1}{N}\,\frac{d_{\rm eff}^2}{\lambda_\omega^3}\,\frac{n_1^\omega+n_2^\omega}{2}\,\frac{T_3^{2\omega}T_1^\omega T_2^\omega}{n_3^{2\omega}n_1^\omega n_2^\omega} \quad ({\rm W}^{-1}\ {\rm m}^{-1}) \quad (1.7.3.56)$$

in the same units as equation (1.7.3.42).

For type I, $n_1^\omega = n_2^\omega$, $T_1^\omega = T_2^\omega$, and for type II $n_1^\omega \neq n_2^\omega$, $T_1^\omega \neq T_2^\omega$.

The attenuation coefficient is written

$$h(L, w_o, \rho, f, \Delta k) = [2z_R(\pi)^{1/2}/L]\int\limits_{-\infty}^{+\infty}|H(a)|^2\exp(-4a^2)\,{\rm d}a$$

with

$$H(a) = \frac{1}{(2\pi)^{1/2}}\int\limits_{-fL/z_R}^{L(1-f)/z_R}\frac{{\rm d}\tau}{1+i\tau}\exp\left[-\gamma^2\left(\tau+\frac{fL}{z_R}\right)^2 - i\sigma\tau\right]$$

$$\text{for type I: } \gamma = 0 \text{ and } \sigma = \Delta k z_R + 4\frac{\rho z_R}{w_o}a$$

$$\text{for type II: } \gamma = \frac{\rho z_R}{w_o(2)^{1/2}} \text{ and } \sigma = \Delta k z_R + 2\frac{\rho z_R}{w_o}a, \quad (1.7.3.57)$$

where f gives the position of the beam waist inside the crystal: $f = 0$ at the entrance and $f = 1$ at the exit surface. The definition and approximations relative to ρ are the same as those discussed for the parallel-beam limit. Δk is the mismatch parameter, which takes into account first a possible shift of the pump beam direction from the collinear phase-matching direction and secondly the distribution of mismatch, including collinear and non-collinear interactions, due to the divergence of the beam, even if the beam axis is phase-matched.