International Tables for Crystallography (2013). Vol. D, Figure 1.7.3.5, p. 196.

Table 1.7.3.6 (*cont.*)

(*c*) SFG type V^4 ($i=1, j=2, k=3$), SFG type VI^4 ($i=2, j=3, k=1$), SFG type VII^4 ($i=3, j=1, k=2$).

Phase-matching loci in the principal planes	Inequalities determining four-wave collinear phase matching in biaxial crystals	
	Positive sign	Negative sign
a	$\frac{n_{x4}}{\lambda_4} < \frac{n_{xi}}{\lambda_i} + \frac{n_{yj}}{\lambda_j} + \frac{n_{yk}}{\lambda_k}; \frac{n_{zi}}{\lambda_i} + \frac{n_{yj}}{\lambda_j} + \frac{n_{yk}}{\lambda_k} < \frac{n_{z4}}{\lambda_4}$	$\frac{n_{yi}}{\lambda_i} + \frac{n_{zj}}{\lambda_j} + \frac{n_{zk}}{\lambda_k} < \frac{n_{y4}}{\lambda_4} < \frac{n_{yi}}{\lambda_i} + \frac{n_{xj}}{\lambda_j} + \frac{n_{xk}}{\lambda_k}$
b	$\frac{n_{yi}}{\lambda_i} + \frac{n_{xj}}{\lambda_j} + \frac{n_{xk}}{\lambda_k} < \frac{n_{y4}}{\lambda_4} < \frac{n_{yi}}{\lambda_i} + \frac{n_{zj}}{\lambda_j} + \frac{n_{zk}}{\lambda_k}$	$\frac{n_{z4}}{\lambda_4} < \frac{n_{zi}}{\lambda_i} + \frac{n_{yj}}{\lambda_j} + \frac{n_{yk}}{\lambda_k}; \frac{n_{xi}}{\lambda_i} + \frac{n_{yj}}{\lambda_j} + \frac{n_{yk}}{\lambda_k} < \frac{n_{x4}}{\lambda_4}$
c′	$\frac{n_{x4}}{\lambda_4} < \frac{n_{xi}}{\lambda_i} + \frac{n_{zj}}{\lambda_j} + \frac{n_{zk}}{\lambda_k}; \frac{n_{yi}}{\lambda_i} + \frac{n_{zj}}{\lambda_j} + \frac{n_{zk}}{\lambda_k} < \frac{n_{y4}}{\lambda_4}$	$\frac{n_{zi}}{\lambda_i} + \frac{n_{yj}}{\lambda_j} + \frac{n_{yk}}{\lambda_k} < \frac{n_{z4}}{\lambda_4} < \frac{n_{zi}}{\lambda_i} + \frac{n_{xj}}{\lambda_j} + \frac{n_{xk}}{\lambda_k}$
*c***	$\frac{n_{xi}}{\lambda_i} + \frac{n_{zj}}{\lambda_j} + \frac{n_{zk}}{\lambda_k} < \frac{n_{x4}}{\lambda_4}; \frac{n_{y4}}{\lambda_4} < \frac{n_{yi}}{\lambda_i} + \frac{n_{zj}}{\lambda_j} + \frac{n_{zk}}{\lambda_k}$	$\frac{n_{zi}}{\lambda_i} + \frac{n_{yj}}{\lambda_j} + \frac{n_{yk}}{\lambda_k} < \frac{n_{z4}}{\lambda_4} < \frac{n_{zi}}{\lambda_i} + \frac{n_{xj}}{\lambda_j} + \frac{n_{xk}}{\lambda_k}$
d′	$\frac{n_{xi}}{\lambda_i} + \frac{n_{yj}}{\lambda_j} + \frac{n_{yk}}{\lambda_k} < \frac{n_{x4}}{\lambda_4} < \frac{n_{xi}}{\lambda_i} + \frac{n_{zj}}{\lambda_j} + \frac{n_{zk}}{\lambda_k}$	$\frac{n_{z4}}{\lambda_4} < \frac{n_{zi}}{\lambda_i} + \frac{n_{xj}}{\lambda_j} + \frac{n_{xk}}{\lambda_k}; \frac{n_{yi}}{\lambda_i} + \frac{n_{xj}}{\lambda_j} + \frac{n_{xk}}{\lambda_k} < \frac{n_{y4}}{\lambda_4}$
*d***	$\frac{n_{xi}}{\lambda_i} + \frac{n_{yj}}{\lambda_j} + \frac{n_{yk}}{\lambda_k} < \frac{n_{x4}}{\lambda_4} < \frac{n_{xi}}{\lambda_i} + \frac{n_{zj}}{\lambda_j} + \frac{n_{zk}}{\lambda_k}$	$\frac{n_{zi}}{\lambda_i} + \frac{n_{xj}}{\lambda_j} + \frac{n_{xk}}{\lambda_k} < \frac{n_{z4}}{\lambda_4}; \frac{n_{y4}}{\lambda_4} < \frac{n_{yi}}{\lambda_i} + \frac{n_{xj}}{\lambda_j} + \frac{n_{xk}}{\lambda_k}$
SFG type V^4, ($i=1$); SFG type VI^4 ($i=2$) ; SFG type VII^4 ($i=3$)		
Conditions *c*′, *d*′ are applied if	$\frac{n_{yi}}{\lambda_i} - \frac{n_{xi}}{\lambda_i} < \frac{n_{y4}}{\lambda_4} - \frac{n_{x4}}{\lambda_4}$	$\frac{n_{yi}}{\lambda_i} - \frac{n_{zi}}{\lambda_i} < \frac{n_{y4}}{\lambda_4} - \frac{n_{z4}}{\lambda_4}$
Conditions *c***, *d*** are applied if	$\frac{n_{y4}}{\lambda_4} - \frac{n_{x4}}{\lambda_4} < \frac{n_{yi}}{\lambda_i} - \frac{n_{xi}}{\lambda_i}$	$\frac{n_{y4}}{\lambda_4} - \frac{n_{z4}}{\lambda_4} < \frac{n_{yi}}{\lambda_i} - \frac{n_{zi}}{\lambda_i}$

π between the waves (Armstrong *et al.*, 1962). This method is called quasi phase matching (QPM). The transfer of energy between the nonlinear polarization and the generated electric field never alternates if the reset is made at each coherence length. In this case and for a three-wave SFG, the nonlinear polarization sequence is the following:

(i) from 0 to L_c, $\mathbf{P}^{NL}(\omega_3) = \varepsilon_0 \chi^{(2)}(\omega_3)\mathbf{e}_1\mathbf{e}_2 E_1 E_2 \exp\{i[k(\omega_1) + k(\omega_2)]Z\}$;

(ii) from L_c to $2L_c$, $\mathbf{P}^{NL}(\omega_3) = -\varepsilon_0 \chi^{(2)}(\omega_3)\mathbf{e}_1\mathbf{e}_2 E_1 E_2 \exp\{i[k(\omega_1) + k(\omega_2)]Z\}$, which is equivalent to $\mathbf{P}^{NL}(\omega_3) = \varepsilon_0 \chi^{(2)}(\omega_3)\mathbf{e}_1\mathbf{e}_2 E_1 E_2 \exp(i\{[k(\omega_1) + k(\omega_2)]Z - \pi\})$.

QPM devices are a recent development and are increasingly being considered for applications (Fejer *et al.*, 1992). The nonlinear medium can be formed by the bonding of thin wafers alternately rotated by π; this has been done for GaAs (Gordon *et al.*, 1993). For ferroelectric crystals, it is possible to form periodic reversing of the spontaneous polarization in the same sample by proton- or ion-exchange techniques, or by applying an electric field, which leads to periodically poled (pp) materials like ppLiNbO$_3$ or ppKTiOPO$_4$ (Myers *et al.*, 1995; Karlsson & Laurell, 1997; Rosenman *et al.*, 1998).

Quasi phase matching offers three main advantages when compared with phase matching: it may be used for any configuration of polarization of the interacting waves, which allows us to use the largest coefficient of the $\chi^{(2)}$ tensor, as explained in the following section; QPM can be achieved over the entire transparency range of the crystal, since the periodicity can be adjusted; and, finally, double refraction and its harmful effect on the nonlinear efficiency can be avoided because QPM can be realized in the principal plane of a uniaxial crystal or in the principal axes of biaxial crystals. Nevertheless, there are limitations due to the difficulty in fabricating the corresponding materials: diffusion-bonded GaAs has strong reflection losses and periodic patterns of ppKTP or ppLN can only be written over a thickness that does not exceed 3 mm, which limits the input energy.

1.7.3.2.4. *Effective coefficient and field tensor*

1.7.3.2.4.1. *Definitions and symmetry properties*

The refractive indices and their dispersion in frequency determine the existence and loci of the phase-matching directions, and so impose the direction of the unit electric field vectors of the interacting waves according to (1.7.3.9). The effective coefficient, given by (1.7.3.23) and (1.7.3.25), depends in part on the linear optical properties *via* the field tensor, which is the tensor product of the interacting unit electric field vectors (Boulanger, 1989; Boulanger & Marnier, 1991; Boulanger *et al.*, 1993; Zyss, 1993). Indeed, the effective coefficient is the contraction between the field tensor and the electric susceptibility tensor of corresponding order:

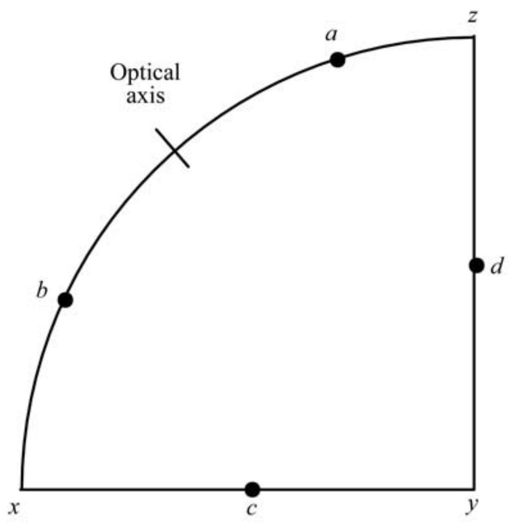

Fig. 1.7.3.5. Stereographic projection on the optical frame of the possible loci of phase-matching directions in the principal planes of a biaxial crystal.

references