International Tables for Crystallography (2013). Vol. D, Section 1.7.3.3.3.2, p. 210.

(n^{ω}, T^{ω}) are relative to the phase-matched SHG crystal and $(n_1^{\omega}, n_2^{2\omega}, n_3^{3\omega}, T_1^{\omega}, T_2^{2\omega}, T_3^{3\omega})$ correspond to the SFG crystal.

In the undepleted pump approximation for SHG, (1.7.3.71) becomes (Qiu & Penzkofer, 1988)

$$\eta_{\rm THG}(L_{\rm SHG}, L_{\rm SFG}) = BT^{\omega}\left[\frac{P^{\omega}(0)}{w_o^2}\right]^2 L_{\rm SHG}^2 L_{\rm SFG}^2 \sin c^2\left(\frac{\Delta k_{\rm SFG} L_{\rm SFG}}{2}\right) \quad (1.7.3.72)$$

with

$$B = B_{\rm SHG}\cdot B_{\rm SFG} = \frac{576\pi^2}{\varepsilon_o^2 c^2}\left(\frac{2N-1}{N}\right)^2 \frac{d^2_{{\rm eff}_{\rm SHG}} d^2_{{\rm eff}_{\rm SFG}}}{\lambda_\omega^4}\left(\frac{T^3_{\rm SHG}}{n^3_{\rm SHG}}\right)\left(\frac{T^3_{\rm SFG}}{n^3_{\rm SFG}}\right)$$

in W^{-2}, where

$$\frac{T^3_{\rm SHG}}{n^3_{\rm SHG}} = \frac{(T^{\omega})^3}{(n^{\omega})^3} \quad \text{and} \quad \frac{T^3_{\rm SFG}}{n^3_{\rm SFG}} = \frac{T_3^{3\omega}T_1^{\omega}T_2^{2\omega}}{n_3^{3\omega}n_1^{\omega}n_2^{2\omega}}.$$

The units are the same as in (1.7.3.42).

A more general case of SFG, where one of the two pump beams is depleted, is given in Section 1.7.3.3.4.

1.7.3.3.3.2. *SHG $(\omega + \omega = 2\omega)$ and SFG $(\omega + 2\omega = 3\omega)$ in the same crystal*

When the SFG conversion efficiency is sufficiently low in comparison with that of the SHG, it is possible to integrate the equations relative to SHG and those relative to SFG separately (Boulanger, Fejer *et al.*, 1994). In order to compare this situation with the example taken for the previous case, we consider a type-I configuration of polarization for SHG. By assuming a perfect phase matching for SHG, the amplitude of the third harmonic field inside the crystal is (Boulanger, 1994)

$$E^{3\omega}(X,Y,Z) = jK^{3\omega}(\varepsilon_o \chi_{{\rm eff}_{\rm SFG}}) \times \int_0^L E^{\omega}_{\rm tot}(X,Y,Z)E^{2\omega}(X,Y,Z)\exp(j\Delta k_{\rm SFG}Z)\,{\rm d}Z \quad (1.7.3.73)$$

with

$$E^{2\omega}(X,Y,Z) = (T^{\omega})^{1/2}|E^{\omega}_{\rm tot}(0)|\tanh(\Gamma Z)$$
$$\text{and} \quad E^{\omega}_{\rm tot}(X,Y,Z) = (T^{\omega})^{1/2}|E^{\omega}_{\rm tot}(0)|\,{\rm sech}(\Gamma Z). \quad (1.7.3.74)$$

Γ is as in (1.7.3.59).

(1.7.3.73) can be analytically integrated for undepleted pump SHG; ${\rm sech}(m) \to 1$, $\tanh(m) \to m$, and so we have

$$\eta_{\rm THG}(L) = P^{3\omega}(L)/P^{\omega}_{\rm tot}(0) \quad (1.7.3.75)$$

with

$$P^{3\omega}(L) = \frac{576\pi^2}{\varepsilon_o^2 c^2}\left(\frac{2N-1}{N}\right)^2 T^{3\omega}\frac{d^2_{{\rm eff}_{\rm SHG}} d^2_{{\rm eff}_{\rm SFG}}}{n^{3\omega}(n^{\omega})^3(n^{2\omega})^2}\frac{[T^{\omega}P^{\omega}_{\rm tot}(0)]^3}{w_o^4\lambda_\omega^4}J(L),$$

where the integral $J(L)$ is

$$J(L) = \left|\int_0^L Z\exp(i\Delta k_{\rm SFG}Z)\,{\rm d}Z\right|^2. \quad (1.7.3.76)$$

For a nonzero SFG phase mismatch, $\Delta k_{\rm SFG} \neq 0$,

$$J(L) \simeq L^2/(\Delta k_{\rm SFG})^2. \quad (1.7.3.77)$$

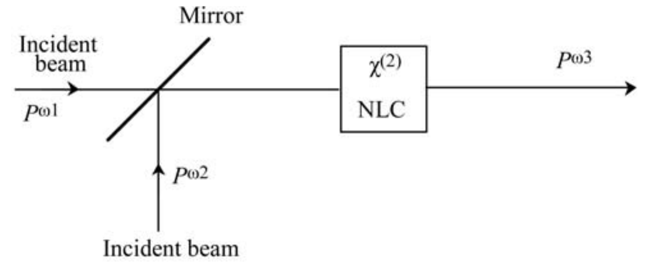

Fig. 1.7.3.17. Frequency up-conversion process $\omega_1 + \omega_2 = \omega_3$. The beam at ω_1 is mixed with the beam at ω_2 in the nonlinear crystal NLC in order to generate a beam at ω_3. $P^{\omega_1,\omega_2,\omega_3}$ are the different powers.

For phase-matched SFG, $\Delta k_{\rm SFG} = 0$,

$$J(L) = L^4/4. \quad (1.7.3.78)$$

Therefore (1.7.3.75) according to (1.7.3.78) is equal to (1.7.3.72) with $L_{\rm SHG} = L_{\rm SFG} = L/2$, $\Delta k_{\rm SFG} = 0$ and 100% transmission coefficients at ω and 2ω between the two crystals.

1.7.3.3.3.3. *Direct THG $(\omega + \omega + \omega = 3\omega)$*

As for the cascading process, we consider a flat plane wave which propagates in a direction without walk-off. The integration of equations (1.7.3.24) over the crystal length L, with $E_4^{3\omega}(X,Y,0) = 0$ and in the undepleted pump approximation, leads to

$$E_4^{3\omega}(X,Y,L) = jK_4^{3\omega}[\varepsilon_o\chi^{(3)}_{\rm eff}]E_1^{\omega}(X,Y,0)E_2^{\omega}(X,Y,0)E_3^{\omega}(X,Y,0) \times L\sin c[(\Delta k\cdot L)/2]\exp(-j\Delta kL/2). \quad (1.7.3.79)$$

According to (1.7.3.36) and (1.7.3.38), the integration of (1.7.3.79) over the cross section, which is the same for the four beams, leads to

$$\eta_{\rm THG}(L) = \frac{P^{3\omega}(L)}{P^{\omega}(0)} = B_{\rm THG}[P^{\omega}(0)]^2\frac{L^2}{w_o^4}\sin c^2[(\Delta k\cdot L)/2]$$

with

$$B_{\rm THG} = \frac{576}{\varepsilon_o^2 c^2}\frac{d^2_{\rm eff}}{\lambda_\omega^2}\frac{T_4^{3\omega}(T_1^{\omega})^2 T_2^{\omega}}{n_4^{3\omega}(n_1^{\omega})^2 n_2^{\omega}} \quad ({\rm m}^2\,{\rm W}^{-2}), \quad (1.7.3.80)$$

where $d_{\rm eff} = (1/4)\chi^{(3)}_{\rm eff}$ is in ${\rm m}^2\,{\rm V}^{-2}$ and λ_ω is in m. The statistical factor is assumed to be equal to 1, which corresponds to a longitudinal single-mode laser.

The different types of phase matching and the associated relations and configurations of polarization are given in Table 1.7.3.2 by considering the SFG case with $\omega_1 = \omega_2 = \omega_3 = \omega_4/3$.

1.7.3.3.4. *Sum-frequency generation (SFG)*

SHG $(\omega + \omega = 2\omega)$ and SFG $(\omega + 2\omega = 3\omega)$ are particular cases of three-wave SFG. We consider here the general situation where the two incident beams at ω_1 and ω_2, with $\omega_1 < \omega_2$, interact with the generated beam at ω_3, with $\omega_3 = \omega_1 + \omega_2$, as shown in Fig. 1.7.3.17. The phase-matching configurations are given in Table 1.7.3.1.

From the general point of view, SFG is a frequency up-conversion parametric process which is used for the conversion of laser beams at low circular frequency: for example, conversion of infrared to visible radiation.

The resolution of system (1.7.3.22) leads to Jacobian elliptic functions if the waves at ω_1 and ω_2 are both depleted. The calculation is simplified in two particular situations which are often encountered: on the one hand undepletion for the waves at ω_1 and ω_2, and on the other hand depletion of only one wave at

references