International Tables for Crystallography (2013). Vol. D, Section 1.7.3.3.5.2, p. 211.

ω_1 or ω_2. For the following, we consider plane waves which propagate in a direction without walk-off so we consider a single wave frame; the energy distribution is assumed to be flat, so the three beams have the same radius w_o.

1.7.3.3.4.1. *SFG* ($\omega_1 + \omega_2 = \omega_3$) *with undepletion at* ω_1 *and* ω_2

The resolution of system (1.7.3.22) with $E_1(X, Y, 0) \neq 0$, $E_2(X, Y, 0) \neq 0$, $\partial E_1(X, Y, Z)/\partial Z = \partial E_2(X, Y, Z)/\partial Z = 0$ and $E_3(X, Y, 0) = 0$, followed by integration over (X, Y), leads to

$$P^{\omega_1}(L) = (T^{\omega_1})^2 P^{\omega_1}(0) \qquad (1.7.3.81)$$

$$P^{\omega_2}(L) = (T^{\omega_2})^2 P^{\omega_2}(0) \qquad (1.7.3.82)$$

$$P^{\omega_3}(L) = BP^{\omega_1}(0)P^{\omega_2}(0)\frac{L^2}{w_o^2}\sin c^2\frac{\Delta k \cdot L}{2} \qquad (1.7.3.83)$$

with

$$B_{\rm SFG} = \frac{72\pi}{\varepsilon_o c}\frac{2N-1}{N}\frac{d_{\rm eff}^2}{\lambda_\omega^2}\frac{T^{\omega_3}T^{\omega_1}T^{\omega_2}}{n^{\omega_3}n^{\omega_1}n^{\omega_2}} \quad ({\rm W}^{-1})$$

in the same units as equation (1.7.3.70).

1.7.3.3.4.2. *SFG* ($\omega_s + \omega_p = \omega_i$) *with undepletion at* ω_p

$(\omega_s, \omega_p, \omega_i) = (\omega_1, \omega_2, \omega_3)$ or $(\omega_2, \omega_1, \omega_3)$.

The undepleted wave at ω_p, the pump, is mixed in the nonlinear crystal with the depleted wave at ω_s, the signal, in order to generate the idler wave at $\omega_i = \omega_s + \omega_p$. The integrations of the coupled amplitude equations over (X, Y, Z) with $E_s(X, Y, 0) \neq 0$, $E_p(X, Y, 0) \neq 0$, $\partial E_p(X, Y, Z)/\partial Z = 0$ and $E_i(X, Y, 0) = 0$ give

$$P_p(L) = T_p^2 P_p(0) \qquad (1.7.3.84)$$

$$P_i(L) = \frac{\omega_i}{\omega_s}P_s(0)\Gamma^2 L^2\frac{\sin^2\{\Gamma^2L^2 + [(\Delta k \cdot L)/2]^2\}^{1/2}}{\Gamma^2L^2 + [(\Delta k \cdot L)/2]^2} \qquad (1.7.3.85)$$

$$P_s(L) = P_s(0)\left[1 - \frac{\omega_s}{\omega_i}\frac{P_i(L)}{P_s(0)}\right], \qquad (1.7.3.86)$$

with $\Delta k = k_i - (k_s + k_p)$ and $\Gamma^2 = [B_s P_p(0)]/w_o^2$, where

$$B_s = \frac{8\pi}{\varepsilon_o c}\frac{2N-1}{N}\frac{d_{\rm eff}^2}{\lambda_s\lambda_i}\frac{T_sT_pT_i}{n_sn_pn_i}.$$

Thus, even if the up-conversion process is phase-matched ($\Delta k = 0$), the power transfers are periodic: the photon transfer efficiency is then 100% for $\Gamma L = (2m+1)(\pi/2)$, where m is an integer, which allows a maximum power gain ω_i/ω_s for the idler. A nonlinear crystal with length $L = (\pi/2\Gamma)$ is sufficient for an optimized device.

For a small conversion efficiency, *i.e.* ΓL weak, (1.7.3.85) and (1.7.3.86) become

$$P_i(L) \simeq P_s(0)\frac{\omega_i}{\omega_s}\Gamma^2L^2\sin c^2\frac{\Delta k \cdot L}{2} \qquad (1.7.3.87)$$

and

$$P_s(L) \simeq P_s(0). \qquad (1.7.3.88)$$

The expression for $P_i(L)$ with $\Delta k = 0$ is then equivalent to (1.7.3.83) with $\omega_p = \omega_1$ or ω_2, $\omega_i = \omega_3$ and $\omega_s = \omega_2$ or ω_1.

For example, the frequency up-conversion interaction can be of great interest for the detection of a signal, ω_s, comprising IR radiation with a strong divergence and a wide spectral bandwidth. In this case, the achievement of a good conversion efficiency, $P_i(L)/P_s(0)$, requires both wide spectral and angular acceptance bandwidths with respect to the signal. The double non-criticality in frequency and angle (DNPM) can then be used with one-beam non-critical non-collinear phase matching (OBNC) associated with vectorial group phase matching (VGPM) (Dolinchuk *et al.*, 1994): this corresponds to the equality of the absolute magnitudes and directions of the signal and idler group velocity vectors *i.e.* $d\omega_i/d\mathbf{k}_i = d\omega_s/d\mathbf{k}_s$.

1.7.3.3.5. *Difference-frequency generation (DFG)*

DFG is defined by $\omega_3 - \omega_1 = \omega_2$ with $E_2(X, Y, 0) = 0$ or $\omega_3 - \omega_2 = \omega_1$ with $E_1(X, Y, 0) = 0$. The DFG phase-matching configurations are given in Table 1.7.3.1. As for SFG, the solutions of system (1.7.3.22) are Jacobian elliptic functions when the incident waves are both depleted. We consider here the simplified situations of undepletion of the two incident waves and depletion of only one incident wave. In the latter, the solutions differ according to whether the circular frequency of the undepleted wave is the highest one, *i.e.* ω_3, or not. We consider the case of plane waves that propagate in a direction without walk-off and we assume a flat energy distribution for the three beams.

1.7.3.3.5.1. *DFG* ($\omega_p - \omega_s = \omega_i$) *with undepletion at* ω_p *and* ω_s

$(\omega_s, \omega_i, \omega_p) = (\omega_1, \omega_2, \omega_3)$ or $(\omega_2, \omega_1, \omega_3)$.

The resolution of system (1.7.3.22) with $E_s(X, Y, 0) \neq 0$, $E_p(X, Y, 0) \neq 0$, $\partial E_p(X, Y, Z)/\partial Z = \partial E_s(X, Y, Z)/\partial Z = 0$ and $E_i(X, Y, 0) = 0$, followed by integration over (X, Y), leads to the same solutions as for SFG with undepletion at ω_1 and ω_2, *i.e.* formulae (1.7.3.81), (1.7.3.82) and (1.7.3.83), by replacing ω_1 by ω_s, ω_2 by ω_p and ω_3 by ω_i. A schematic device is given in Fig. 1.7.3.17 by replacing $(\omega_1, \omega_2, \omega_3)$ by $(\omega_1, \omega_3, \omega_2)$ or $(\omega_2, \omega_3, \omega_1)$.

1.7.3.3.5.2. *DFG* ($\omega_s - \omega_p = \omega_i$) *with undepletion at* ω_p

$(\omega_s, \omega_i, \omega_p) = (\omega_3, \omega_1, \omega_2)$ or $(\omega_3, \omega_2, \omega_1)$.

The resolution of system (1.7.3.22) with $E_s(X, Y, 0) \neq 0$, $E_p(X, Y, 0) \neq 0$, $\partial E_p(X, Y, Z)/\partial Z = 0$ and $E_i(X, Y, 0) = 0$, followed by the integration over (X, Y), leads to the same solutions as for SFG with undepletion at ω_1 or ω_2: formulae (1.7.3.84), (1.7.3.85) and (1.7.3.86).

1.7.3.3.5.3. *DFG* ($\omega_p - \omega_s = \omega_i$) *with undepletion at* ω_p *– optical parametric amplification (OPA), optical parametric oscillation (OPO)*

$(\omega_s, \omega_i, \omega_p) = (\omega_1, \omega_2, \omega_3)$ or $(\omega_2, \omega_1, \omega_3)$.

The initial conditions are the same as in Section 1.7.3.3.5.2, except that the undepleted wave has the highest circular frequency. In this case, the integrations of the coupled amplitude equations over (X, Y, Z) lead to

$$P_p(L) = T_p^2 P_p(0), \qquad (1.7.3.89)$$

$$P_i(L) = P_s(0)\frac{\omega_i}{\omega_s}\Gamma^2L^2\frac{\sinh^2\{\Gamma^2L^2 - [(\Delta k \cdot L)/2]^2\}^{1/2}}{\Gamma^2L^2 - [(\Delta k \cdot L)/2]^2} \qquad (1.7.3.90)$$

and

$$P_s(L) = P_s(0)\left[1 + \frac{\omega_s}{\omega_i}\frac{P_i(L)}{P_s(0)}\right] = P_s(0)\left(1 + \Gamma^2L^2\frac{\sinh^2\{\Gamma^2L^2 - [(\Delta k \cdot L)/2]^2\}^{1/2}}{\Gamma^2L^2 - [(\Delta k \cdot L)/2]^2}\right) \qquad (1.7.3.91)$$

with $\Delta k = k_p - (k_i + k_s)$ and $\Gamma^2 = [B_iP_p(0)]/w_o^2$, where w_o is the beam radius of the three beams and

$$B_i = \frac{8\pi}{\varepsilon_o c}\frac{2N-1}{N}\frac{d_{\rm eff}^2}{\lambda_s\lambda_i}\frac{T_sT_pT_i}{n_sn_pn_i}.$$

  references