

1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

components for the other configurations of polarization are obtained by permutation of the Cartesian indices and the corresponding polarizations.

From Tables 1.7.3.7 and 1.7.3.8, it is possible to deduce all the other *2e.o* interactions (*eeo*), (*oeo*), the *2o.e* interactions (*ooe*), (*oeo*), the *3o.e* interactions (*oooe*), (*oeoo*), (*ooeo*), the *3e.o* interactions (*eoee*), (*eeoe*), (*eeeo*) and the *2o.2e* interactions (*oeoe*), (*eeoo*), (*eeoo*), (*oeeo*), (*eeoe*). The corresponding interactions and types are given in Tables 1.7.3.1 and 1.7.3.2. According to (1.7.3.31) and (1.7.3.33), the magnitudes of two permuted components are equal if the permutation of polarizations are associated with the corresponding frequencies. For example, according to Table 1.7.3.2, two permuted field-tensor components have the same magnitude for permutation between the following *3o.e* interactions:

(i) (*eoee*) SFG (ω_4) type I < 0 and the three (*ooee*) interactions, DFG (ω_1) type II < 0, DFG (ω_2) type III < 0, DFG (ω_3) type IV < 0;

(ii) the three (*ooee*) interactions, SFG (ω_4) type II > 0, DFG (ω_1) type III > 0, DFG (ω_2) type IV > 0 and (*eoee*) DFG (ω_3) type I > 0;

(iii) the two (*ooee*) interactions SFG (ω_4) type III > 0, DFG (ω_1) type IV > 0, (*eoee*) DFG (ω_2) type I > 0, and (*ooee*) DFG (ω_3) type II > 0;

(iv) (*eeoo*) SFG (ω_4) type IV > 0, (*eoee*) DFG (ω_1) type I > 0, and the two interactions (*ooee*) DFG (ω_2) type II > 0, DFG (ω_3) type III > 0.

The contraction of the field tensor and the uniaxial dielectric susceptibility tensor of corresponding order, given in Tables 1.7.2.2 to 1.7.2.5, is nil for the following uniaxial crystal classes

Table 1.7.3.7. Matrix representations of the (*oeo*) and (*ooo*) field tensors of the uniaxial class and of the biaxial class in the principal planes *xz* and *yz*, with $\omega_1 \neq \omega_2$ (Boulanger & Marnier, 1991)

$\bullet \quad F_{ijk} = 0$ $\bullet \text{---} \bullet \quad , \quad \circ \text{---} \circ \quad F_{ijk} = F_{lmn}$ $\bullet \text{---} \circ \quad F_{ijk} = -F_{lmn}$

Interactions	Three-rank $F_{ijk}(\theta, \varphi)$ field tensors
Type <i>ooo</i> SFG (ω_3) type I < 0 DFG (ω_1) type I > 0 DFG (ω_2) type I > 0	
Type <i>oeo</i> SFG (ω_3) type I > 0 DFG (ω_1) type I < 0 DFG (ω_2) type I < 0	

Table 1.7.3.8. Matrix representations of the (*oeee*), (*eoee*) and (*ooee*) field tensors of the uniaxial class and of the biaxial class in the principal planes *xz* and *yz*, with $\omega_1 \neq \omega_2 \neq \omega_3$ (Boulanger et al., 1993)

$\bullet \quad F_{ijkl} = 0$ $\bullet \text{---} \bullet \quad F_{ijkl} = F_{mnop}$ $\bullet \text{---} \circ \quad F_{ijkl} = -F_{mnop}$

Interactions	Four-rank $F_{ijkl}(\theta, \varphi)$ field tensors
Type <i>oeee</i> SFG(ω_4) type I > 0 DFG (ω_1) type I < 0 DFG (ω_2) type I < 0 DFG (ω_3) type I < 0	
Type <i>eoee</i> SFG (ω_4) type I < 0 DFG (ω_1) type I > 0 DFG (ω_2) type I > 0 DFG (ω_3) type I > 0	
Type <i>ooee</i> SFG (ω_4) type $V^4 > 0$ DFG (ω_1) type $V^1 > 0$ DFG (ω_2) type $V^2 > 0$ DFG (ω_3) type $V^3 > 0$	