Table 1.7.3.9. *Field-tensor components specifically nil in the principal planes of uniaxial and biaxial crystals for three-wave and four-wave interactions* $(i, j, k) = x, y$ or z.

Configurations of polarization	Nil field-tensor components		
	(xy) plane	(xz) plane	(yz) plane
eoo	$F_{xjk} = 0$; $F_{yjk} = 0$	$F_{ixk} = F_{ijx} = 0$ $F_{yjk} = 0$	$F_{iyk} = F_{ijy} = 0$ $F_{xjk} = 0$
oee	$F_{ixk} = F_{ijx} = 0$ $F_{iyk} = F_{ijy} = 0$	$F_{iyk} = F_{ijy} = 0$ $F_{xik} = 0$	$F_{ixk} = F_{ijx} = 0$ $F_{yjk} = 0$
eooo	$F_{xjkl} = 0$; $F_{yjkl} = 0$	$F_{ixkl} = F_{ijxl} = F_{ijkx} = 0$ $F_{yjkl} = 0$	$F_{iykl} = F_{ijyl} = F_{ijky} = 0$ $F_{xjkl} = 0$
oeee	$F_{ixkl} = F_{ijxl} = F_{ijkx} = 0$ $F_{iykl} = F_{ijyl} = F_{ijky} = 0$	$F_{iykl} = F_{ijyl} = F_{ijky} = 0$ $F_{xjkl} = 0$	$F_{ixkl} = F_{ijxl} = F_{ijkx} = 0$ $F_{yjkl} = 0$
ooee	$F_{ijxl} = F_{ijkx} = 0$ $F_{ijyl} = F_{ijky} = 0$	$F_{xjkl} = F_{ixkl} = 0$ $F_{ijyl} = F_{ijky} = 0$	$F_{yjkl} = F_{iykl} = 0$ $F_{ijxl} = F_{ijkx} = 0$

and configurations of polarization: D_4 and D_6 for $2o.e$, C_{4v} and C_{6v} for $2e.o$, D_6, D_{6h}, D_{3h} and C_{6v} for $3o.e$ and $3e.o$. Thus, even if phase-matching directions exist, the effective coefficient in these situations is nil, which forbids the interactions considered (Boulanger & Marnier, 1991; Boulanger *et al.*, 1993). The number of forbidden crystal classes is greater under the Kleinman approximation. The forbidden crystal classes have been determined for the particular case of third harmonic generation assuming Kleinman conjecture and without consideration of the field tensor (Midwinter & Warner, 1965).

1.7.3.2.4.3. *Biaxial class*

The symmetry of the biaxial field tensors is the same as for the uniaxial class, though only for a propagation in the principal planes xz and yz; the associated matrix representations are given in Tables 1.7.3.7 and 1.7.3.8, and the nil components are listed in Table 1.7.3.9. Because of the change of optic sign from either side of the optic axis, the field tensors of the interactions for which the phase-matching cone joins areas b and a or a and c, given in Fig. 1.7.3.5, change from one area to another: for example, the field tensor (*eoee*) becomes an (*oeoo*) and so the solicited components of the electric susceptibility tensor are not the same.

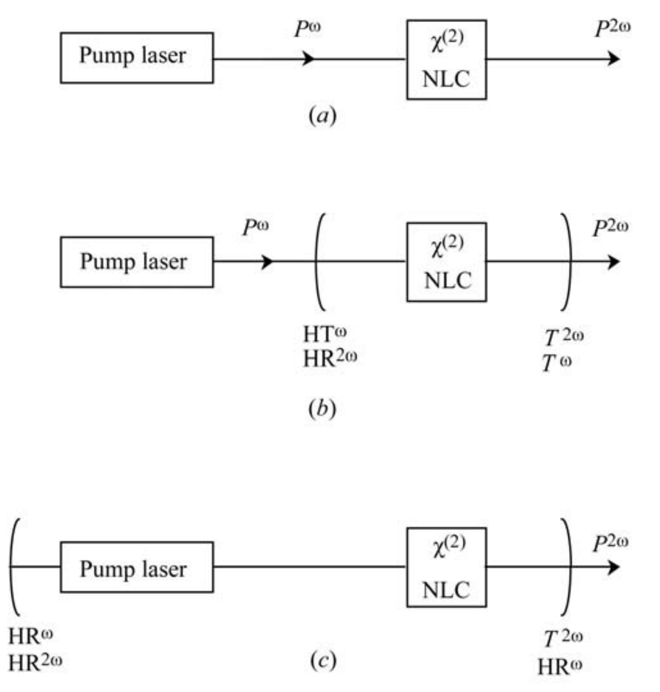

Fig. 1.7.3.6. Schematic configurations for second harmonic generation. (*a*) Non-resonant SHG; (*b*) external resonant SHG: the resonant wave may either be the fundamental or the harmonic one; (*c*) internal resonant SHG. $P^{\omega,2\omega}$ are the fundamental and harmonic powers; HT^{ω} and $HR^{\omega,2\omega}$ are the high-transmission and high-reflection mirrors at ω or 2ω and $T^{\omega,2\omega}$ are the transmission coefficients of the output mirror at ω or 2ω. NLC is the nonlinear crystal with a nonzero $\chi^{(2)}$.

The nonzero field-tensor components for a propagation in the xy plane of a biaxial crystal are: F_{zxx}, F_{zyy}, $F_{zxy} \neq F_{zyx}$ for (*eoo*); F_{xzz}, F_{yzz} for (*oee*); F_{zxxx}, F_{zyyy}, $F_{zxyy} \neq F_{zyxy} \neq F_{zyyx}$, $F_{zxxy} \neq F_{zxyx} \neq F_{zyxx}$ for (*eooo*); F_{xzzz}, F_{yzzz} for (*oeee*); $F_{xyzz} \neq F_{yxzz}$, F_{xxzz}, F_{yyzz} for (*ooee*). The nonzero components for the other configurations of polarization are obtained by the associated permutations of the Cartesian indices and the corresponding polarizations.

The field tensors are not symmetric for a propagation out of the principal planes in the general case where all the frequencies are different: in this case there are 27 independent components for the three-wave interactions and 81 for the four-wave interactions, and so all the electric susceptibility tensor components are solicited.

As phase matching imposes the directions of the electric fields of the interacting waves, it also determines the field tensor and hence the effective coefficient. Thus there is no possibility of choice of the $\chi^{(2)}$ coefficients, since a given type of phase matching is considered. In general, the largest coefficients of polar crystals, *i.e.* χ_{zzz}, are implicated at a very low level when phase matching is achieved, because the corresponding field tensor, *i.e.* F_{zzz}, is often weak (Boulanger *et al.*, 1997). In contrast, QPM authorizes the coupling between three waves polarized along the z axis, which leads to an effective coefficient which is purely χ_{zzz}, *i.e.* $\chi_{\text{eff}} = (2/\pi)\chi_{zzz}$, where the numerical factor comes from the periodic character of the rectangular function of modulation (Fejer *et al.*, 1992).

1.7.3.3. *Integration of the propagation equations*

1.7.3.3.1. *Spatial and temporal profiles*

The resolution of the coupled equations (1.7.3.22) or (1.7.3.24) over the crystal length L leads to the electric field amplitude $E_i(X, Y, L)$ of each interacting wave. The general solutions are Jacobian elliptic functions (Armstrong *et al.*, 1962; Fève, Boulanger & Douady, 2002). The integration of the systems is simplified for cases where one or several beams are held constant, which is called the undepleted pump approximation. We consider mainly this kind of situation here. The power of each interacting wave is calculated by integrating the intensity over the cross section of each beam according to (1.7.3.8). For our main purpose, we consider the simple case of plane-wave beams with two kinds of transverse profile:

$$\begin{aligned} \mathbf{E}(X, Y, Z) &= \mathbf{e}E_o(Z) \quad \text{for } (X, Y) \in [-w_o, +w_o] \\ \mathbf{E}(X, Y, Z) &= 0 \quad \text{elsewhere} \end{aligned} \qquad (1.7.3.36)$$

for a flat distribution over a radius w_o;

$$\mathbf{E}(X, Y, Z) = \mathbf{e}E_o(Z)\exp[-(X^2 + Y^2)/w_o^2] \qquad (1.7.3.37)$$

for a Gaussian distribution, where w_o is the radius at $(1/e)$ of the electric field and so at $(1/e^2)$ of the intensity.

references