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dissipation theorem in the classical limit for $h\delta\nu \ll k_BT$ (Hayes & Loudon, 1978). The coupling coefficient M is given by

$$M = |e_m e'_n \kappa_{mi} \kappa_{nj} p'_{ijk\ell} \hat{u}_k \hat{Q}_\ell|^2. \qquad (2.4.4.8)$$

In practice, the incident intensity is defined outside the scattering volume, $I_{\rm out}$, and for normal incidence one can write

$$I_{\rm in} = \frac{4n}{(n+1)^2} I_{\rm out}. \qquad (2.4.4.9a)$$

Similarly, the scattered power is observed outside as $P_{\rm out}$, and

$$P_{\rm out} = \frac{4n'}{(n'+1)^2} P_{\rm in}, \qquad (2.4.4.9b)$$

again for normal incidence. Finally, the approximative relation between the scattering solid angle $\Omega_{\rm out}$, outside the sample, and the solid angle $\Omega_{\rm in}$, in the sample, is

$$\Omega_{\rm out} = (n')^2 \Omega_{\rm in}. \qquad (2.4.4.9c)$$

Substituting (2.4.4.9*a*,*b*,*c*) in (2.4.4.7), one obtains (Vacher & Boyer, 1972)

$$\frac{{\rm d}P_{\rm out}}{{\rm d}\Omega_{\rm out}} = \frac{8\pi^2 k_B T}{\lambda_0^4} \frac{n^4}{(n+1)^2} \frac{(n')^4}{(n'+1)^2} \beta V I_{\rm out}, \qquad (2.4.4.10)$$

where the coupling coefficient β is

$$\beta = \frac{1}{n^4 (n')^4} \frac{|e_m e'_n \kappa_{mi} \kappa_{nj} p'_{ijk\ell} \hat{u}_k \hat{Q}_\ell|^2}{C}. \qquad (2.4.4.11)$$

In the cases of interest here, the tensor $\boldsymbol{\kappa}$ is diagonal, $\kappa_{ij} = n_i^2 \delta_{ij}$ without summation on i, and (2.4.4.11) can be written in the simpler form

$$\beta = \frac{1}{n^4 (n')^4} \frac{|e_i n_i^2 p'_{ijk\ell} \hat{u}_k \hat{Q}_\ell e'_j n_j^2|^2}{C}. \qquad (2.4.4.12)$$

2.4.5. Use of the tables

The tables in this chapter give information on modes and scattering geometries that are in most common use in the study of hypersound in single crystals. Just as in the case of X-rays, Brillouin scattering is not sensitive to the presence or absence of a centre of symmetry (Friedel, 1913). Hence, the results are the same for all crystalline classes belonging to the same centric group, also called Laue class. The correspondence between the point groups and the Laue classes analysed here is shown in Table 2.4.5.1. The monoclinic and triclinic cases, being too cumbersome, will not be treated here.

For tensor components $c_{ijk\ell}$ and $p_{ijk\ell}$, the tables make use of the usual contracted notation for index pairs running from 1 to 6. However, as the tensor $p'_{ijk\ell}$ is not symmetric upon interchange of (k, ℓ), it is necessary to distinguish the order (k, ℓ) and (ℓ, k). This is accomplished with the following correspondence:

$$\begin{array}{lll} 1,1 \to 1 & 2,2 \to 2 & 3,3 \to 3 \\ 1,2 \to 6 & 2,3 \to 4 & 3,1 \to 5 \\ 2,1 \to \bar{6} & 3,2 \to \bar{4} & 1,3 \to \bar{5}. \end{array}$$

Geometries for longitudinal modes (LA) are listed in Tables 2.4.5.2 to 2.4.5.8. The first column gives the direction of the scattering vector $\hat{\mathbf{Q}}$ that is parallel to the displacement $\hat{\mathbf{u}}$. The second column gives the elastic coefficient according to (2.4.2.6). In piezoelectric materials, effective elastic coefficients defined in (2.4.2.11) must be used in this column. The third column gives the direction of the light polarizations $\hat{\mathbf{e}}$ and $\hat{\mathbf{e}}'$, and the last column gives the corresponding coupling coefficient β [equation (2.5.5.11)]. In general, the strongest scattering intensity is obtained for polarized scattering ($\hat{\mathbf{e}} = \hat{\mathbf{e}}'$), which is the only situation listed in the tables. In this case, the coupling to light (β) is independent of the scattering angle θ, and thus the tables apply to any θ value.

Tables 2.4.5.9 to 2.4.5.15 list the geometries usually used for the observation of TA modes in backscattering ($\theta = 180^\circ$). In this case, $\hat{\mathbf{u}}$ is always perpendicular to $\hat{\mathbf{Q}}$ (pure transverse modes), and $\hat{\mathbf{e}}'$ is not necessarily parallel to $\hat{\mathbf{e}}$. Cases where pure TA modes with $\hat{\mathbf{u}}$ in the plane perpendicular to $\hat{\mathbf{Q}}$ are degenerate are indicated by the symbol D in the column for $\hat{\mathbf{u}}$. For the Pockels tensor components, the notation is $p_{\alpha\beta}$ if the rotational term vanishes by symmetry, and it is $p'_{\alpha\beta}$ otherwise.

Tables 2.4.5.16 to 2.4.5.22 list the common geometries used for the observation of TA modes in 90° scattering. In these tables, the polarization vector $\hat{\mathbf{e}}$ is always perpendicular to the scattering plane and $\hat{\mathbf{e}}'$ is always parallel to the incident wavevector of light $\mathbf{q}$. Owing to birefringence, the scattering vector $\hat{\mathbf{Q}}$ does not exactly bisect $\mathbf{q}$ and $\mathbf{q}'$ [equation (2.4.4.4)]. The tables are written for strict 90° scattering, $\mathbf{q} \cdot \mathbf{q}' = 0$, and in the case of birefringence the values of $\mathbf{q}^{(m)}$ to be used are listed separately in Table 2.4.5.23. The latter assumes that the birefringences are not large, so that the values of $\mathbf{q}^{(m)}$ are given only to first order in the birefringence.

2.4.6. Techniques of Brillouin spectroscopy

Brillouin spectroscopy with visible laser light requires observing frequency shifts falling typically in the range ~1 to ~100 GHz, or ~0.03 to ~3 cm^{-1}. To achieve this with good resolution one mostly employs interferometry. For experiments at very small angles (near forward scattering), photocorrelation spectroscopy can also be used. If the observed frequency shifts are ≥ 1 cm^{-1}, rough measurements of spectra can sometimes be obtained with modern grating instruments. Recently, it has also become possible to perform Brillouin scattering using other excitations, in particular neutrons or X-rays. In these cases, the coupling does not occur *via* the Pockels effect, and the frequency shifts that are observed are much larger. The following discussion is restricted to optical interferometry.

The most common interferometer that has been used for this purpose is the single-pass planar Fabry–Perot (Born & Wolf, 1993). Upon illumination with monochromatic light, the frequency response of this instrument is given by the Airy function, which consists of a regular comb of maxima obtained as the optical path separating the mirrors is increased. Successive maxima are separated by $\lambda/2$. The ratio of the maxima separation to the width of a single peak is called the finesse F, which increases as the mirror reflectivity increases. The finesse is also limited by the planarity of the mirrors. A practical limit is $F \sim 100$. The resolving power of such an instrument is $R = 2\ell/\lambda$, where ℓ is the optical thickness. Values of R around 10^6 to 10^7 can be achieved. It is impractical to increase ℓ above ~5 cm because the luminosity of the instrument is proportional to $1/\ell$. If higher

Table 2.4.5.1. *Definition of Laue classes*

Crystal system	Laue class	Point groups
Cubic	C_1	$432, \bar{4}3m, m\bar{3}m$
	C_2	$23, \bar{3}m$
Hexagonal	H_1	$622, 6mm, \bar{6}2m, 6/mm$
	H_2	$6, \bar{6}, 6/m$
Tetragonal	T_1	$422, 4mm, \bar{4}2m, 4/mm$
	T_2	$4, \bar{4}, 4/m$
Trigonal	R_1	$32, 3m, \bar{3}m$
	R_2	$3, \bar{3}$
Orthorhombic	O	$mmm, 2mm, 222$

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Table 2.4.5.2. *Cubic Laue classes C_1 and C_2: longitudinal modes*

This table, written for the class C_2, is also valid for the class C_1 with the additional relation $p_{12} = p_{13}$. It can also be used for the spherical system where $c_{44} = \frac{1}{2}(c_{11} - c_{12})$, $p_{44} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	C	$\mathbf{e} = \mathbf{e}'$	β
(1, 0, 0)	c_{11}	(0, 1, 0)	p_{13}^2/c_{11}
(1, 0, 0)	c_{11}	(0, 0, 1)	p_{12}^2/c_{11}
$(1, 1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} + c_{12}) + c_{44}$	(0, 0, 1)	$(p_{12} + p_{13})^2/4C$
$(1, 1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} + c_{12}) + c_{44}$	$(1, -1, 0)/\sqrt{2}$	$(2p_{11} + p_{12} + p_{13} - 4p_{44})^2/16C$
$(1, 1, 1)/\sqrt{3}$	$\frac{1}{3}(c_{11} + 2c_{12} + 4c_{44})$	$(1, 1, -2)/\sqrt{6}$	$(p_{11} + p_{12} + p_{13} - 2p_{44})^2/9C$
$(1, 1, 1)/\sqrt{3}$	$\frac{1}{3}(c_{11} + 2c_{12} + 4c_{44})$	$(1, -1, 0)/\sqrt{2}$	$(p_{11} + p_{12} + p_{13} - 2p_{44})^2/9C$

Table 2.4.5.3. *Tetragonal T_1 and hexagonal H_1 Laue classes: longitudinal modes*

This table, written for the class T_1, is also valid for the class H_1 with the additional relations $c_{66} = \frac{1}{2}(c_{11} - c_{12})$; $p_{66} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	C	$\mathbf{e} = \mathbf{e}'$	β
(1, 0, 0)	c_{11}	(0, 1, 0)	p_{12}^2/c_{11}
(1, 0, 0)	c_{11}	(0, 0, 1)	p_{31}^2/c_{11}
(0, 0, 1)	c_{33}	(1, 0, 0)	p_{13}^2/c_{33}
(0, 0, 1)	c_{33}	(0, 1, 0)	p_{13}^2/c_{33}
$(1, 1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} + c_{12}) + c_{66}$	(0, 0, 1)	p_{31}^2/C
$(1, 1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} + c_{12}) + c_{66}$	$(1, -1, 0)/\sqrt{2}$	$(p_{11} + p_{12} - 2p_{66})^2/4C$

Table 2.4.5.4. *Hexagonal Laue class H_2: longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	C	$\mathbf{e} = \mathbf{e}'$	β
(1, 0, 0)	c_{11}	(0, 1, 0)	p_{12}^2/c_{11}
(1, 0, 0)	c_{11}	(0, 0, 1)	p_{31}^2/c_{11}
(0, 0, 1)	c_{33}	(1, 0, 0)	p_{13}^2/c_{33}
(0, 0, 1)	c_{33}	(0, 1, 0)	p_{13}^2/c_{33}
$(1, 1, 0)/\sqrt{2}$	c_{11}	(0, 0, 1)	p_{31}^2/c_{11}
$(1, 1, 0)/\sqrt{2}$	c_{11}	$(1, -1, 0)/\sqrt{2}$	p_{12}^2/c_{11}

Table 2.4.5.5. *Tetragonal Laue class T_2: longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	C	$\mathbf{e} = \mathbf{e}'$	β
(0, 0, 1)	c_{33}	(1, 0, 0)	p_{13}^2/c_{33}
(0, 0, 1)	c_{33}	(0, 1, 0)	p_{13}^2/c_{33}

Table 2.4.5.6. *Orthorhombic Laue class O: longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	C	$\mathbf{e} = \mathbf{e}'$	β
(1, 0, 0)	c_{11}	(0, 1, 0)	p_{21}^2/c_{11}
(1, 0, 0)	c_{11}	(0, 0, 1)	p_{31}^2/c_{11}
(0, 1, 0)	c_{22}	(0, 0, 1)	p_{32}^2/c_{22}
(0, 1, 0)	c_{22}	(1, 0, 0)	p_{12}^2/c_{22}
(0, 0, 1)	c_{33}	(1, 0, 0)	p_{13}^2/c_{33}
(0, 0, 1)	c_{33}	(0, 1, 0)	p_{23}^2/c_{33}

Table 2.4.5.7. *Trigonal Laue class R_1: longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
(1, 0, 0)	c_{11}	(0, 1, 0)	(0, 1, 0)	p_{12}^2/c_{11}
(1, 0, 0)	c_{11}	(0, 0, 1)	(0, 0, 1)	p_{31}^2/c_{11}
(1, 0, 0)	c_{11}	(0, 1, 0)	(0, 0, 1)	p_{41}^2/c_{11}
(0, 0, 1)	c_{33}	(1, 0, 0)	(1, 0, 0)	p_{13}^2/c_{33}
(0, 0, 1)	c_{33}	(0, 1, 0)	(0, 1, 0)	p_{13}^2/c_{33}

Table 2.4.5.8. *Trigonal Laue class R_2: longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
(0, 0, 1)	c_{33}	(1, 0, 0)	(1, 0, 0)	p_{13}^2/c_{33}
(0, 0, 1)	c_{33}	(0, 1, 0)	(0, 1, 0)	p_{13}^2/c_{33}

resolutions are required, one uses a spherical interferometer as described below.

A major limitation of the Fabry–Perot interferometer is its poor contrast, namely the ratio between the maximum and the minimum of the Airy function, which is typically $\sim$1000. This limits the use of this instrument to samples of very high optical quality, as otherwise the generally weak Brillouin signals are masked by the elastically scattered light. To avert this effect, several passes are made through the same instrument, thus elevating the Airy function to the corresponding power (Hariharan & Sen, 1961; Sandercock, 1971). Multiple-pass instruments with three, four or five passes are common. Another limitation of the standard Fabry–Perot interferometer is that the interference pattern is repeated at each order. Hence, if the spectrum has a broad spectral spread, the overlap of adjacent orders can greatly complicate the interpretation of measurements. In this case, tandem instruments can be of considerable help. They consist of two Fabry–Perot interferometers with combs of different periods placed in series (Chantrel, 1959; Mach *et al.*, 1963). These are operated around a position where the peak transmission of the first interferometer coincides with that of the second one. The two Fabry–Perot interferometers are scanned simultaneously. With this setup, the successive orders are reduced to small ghosts and overlap is not a problem. A convenient commercial instrument has been designed by Sandercock (1982).

To achieve higher resolutions, one uses the spherical Fabry–Perot interferometer (Connes, 1958; Hercher, 1968). This consists of two spherical mirrors placed in a near-confocal configuration. Their spacing ℓ is scanned over a distance of the order of λ. The peculiarity of this instrument is that its luminosity increases with its resolution. One obvious drawback is that a change of resolving power, *i.e.* of ℓ, requires other mirrors. Of course, the single spherical Fabry–Perot interferometer suffers the same limitations regarding contrast and order overlap that were discussed above for the planar case. Multipassing the spherical Fabry–Perot interferometer is possible but not very convenient. It is preferable to use tandem instruments that combine a multipass planar instrument of low resolution followed by a spherical instrument of high resolution (Pine, 1972; Vacher, 1972). To analyse the linewidth of narrow phonon lines, the planar standard is adjusted dynamically to transmit the Brillouin line and the spherical interferometer is scanned across the line. With such a device, resolving powers of $\sim 10^8$ have been achieved. For the dynamical adjustment of this instrument one can use a reference signal near the frequency of the phonon line, which is derived by electro-optic modulation of the exciting laser (Sussner & Vacher, 1979). In this case, not only the width of the phonon, but also its absolute frequency shift, can be determined with an accuracy of $\sim$1 MHz. It is obvious that to achieve this kind of resolution, the laser source itself must be appropriately stabilized.

In closing, it should be stressed that the practice of interferometry is still an art that requires suitable skills and training in spite of the availability of commercial instruments. The experimenter must take care of a large number of aspects relating to the

Table 2.4.5.9. *Cubic Laue classes C_1 and C_2: transverse modes, backscattering*

This table, written for the class C_2, is also valid for the class C_1 with the additional relation $p_{12} = p_{13}$. It can also be used for the spherical system where $c_{44} = \frac{1}{2}(c_{11} - c_{12})$, $p_{44} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
$(1, 1, 0)/\sqrt{2}$	$(1, -1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} - c_{12})$	$(0, 0, 1)$	$(0, 0, 1)$	$(p_{12} - p_{13})^2/2(c_{11} - c_{12})$
$(1, 1, 1)/\sqrt{3}$	D	$\frac{1}{3}(c_{11} - c_{12} + c_{44})$	$(1, 1, -2)/\sqrt{6}$	$(1, -1, 0)/\sqrt{2}$	$[3(p_{12} - p_{13})^2 + (p_{12} + p_{13} + 4p_{44} - 2p_{11})^2]/72C$

Table 2.4.5.10. *Tetragonal T_1 and hexagonal H_1 Laue classes: transverse modes, backscattering*

This table, written for the class T_1, is also valid for the class H_1 with the additional relations $c_{66} = \frac{1}{2}(c_{11} - c_{12})$; $p_{66} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66})$	$(1, 0, 0)$	$(0, 1, -1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C](n_1^2p_{66} - n_3^2p'_{44})^2$

Table 2.4.5.11. *Hexagonal Laue class H_2: transverse modes, backscattering*

$c_{66} = \frac{1}{2}(c_{11} - c_{12})$; $p_{66} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
$(1, 0, 0)$	$(0, 1, 0)$	c_{66}	$(0, 1, 0)$	$(0, 1, 0)$	p_{16}^2/c_{66}
$(1, 0, 0)$	$(0, 0, 1)$	c_{44}	$(0, 1, 0)$	$(0, 0, 1)$	p_{45}^2/c_{44}
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66})$	$(1, 0, 0)$	$(1, 0, 0)$	$p_{16}^2/(c_{44} + c_{66})$
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66})$	$(1, 0, 0)$	$(0, 1, -1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C](n_1^2p_{66} - n_3^2p'_{44})^2$

Table 2.4.5.12. *Tetragonal Laue class T_2: transverse modes, backscattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
$(1, 0, 0)$	$(0, 0, 1)$	c_{44}	$(0, 1, 0)$	$(0, 0, 1)$	p_{45}^2/c_{44}
$(1, 1, 0)/\sqrt{2}$	$(0, 0, 1)$	c_{44}	$(0, 0, 1)$	$(1, -1, 0)/\sqrt{2}$	p_{45}^2/c_{44}

Table 2.4.5.13. *Orthorhombic Laue class O: transverse modes, backscattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
$(1, 1, 0)/\sqrt{2}$	$(0, 0, 1)$	$\frac{1}{2}(c_{44} + c_{55})$	$(0, 0, 1)$	$(1, -1, 0)/\sqrt{2}$	$[(n_1^2 + n_2^2)^2/16n_1^4n_2^4C](n_1^2p'_{55} - n_2^2p'_{4\bar{4}})^2$
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{55} + c_{66})$	$(1, 0, 0)$	$(0, 1, -1)/\sqrt{2}$	$[(n_2^2 + n_3^2)^2/16n_2^4n_3^4C](n_2^2p'_{66} - n_3^2p'_{5\bar{5}})^2$
$(1, 0, 1)/\sqrt{2}$	$(0, 1, 0)$	$\frac{1}{2}(c_{44} + c_{66})$	$(0, 1, 0)$	$(-1, 0, 1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C](n_3^2p'_{44} - n_1^2p'_{6\bar{6}})^2$

Table 2.4.5.14. *Trigonal Laue class R_1: transverse modes, backscattering*

$c_{66} = \frac{1}{2}(c_{11} - c_{12})$; $p_{66} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
$(0, 1, 0)$	$(1, 0, 0)$	c_{66}	$(0, 0, 1)$	$(1, 0, 0)$	p_{41}^2/c_{66}
$(0, 0, 1)$	D	c_{44}	$(1, 0, 0)$	$(1, 0, 0)$	p_{14}^2/c_{44}
$(0, 0, 1)$	D	c_{44}	$(0, 1, 0)$	$(1, 0, 0)$	p_{14}^2/c_{44}
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66}) + c_{14}$	$(1, 0, 0)$	$(0, 1, -1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C][n_1^2(p_{66} + p_{14}) - n_3^2(p'_{44} + p_{41})]^2$
$(0, 1, -1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66}) - c_{14}$	$(1, 0, 0)$	$(0, 1, 1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C][n_1^2(p_{66} - p_{14}) + n_3^2(p_{41} - p'_{44})]^2$

Table 2.4.5.15. *Trigonal Laue class R_2: transverse modes, backscattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	$\mathbf{e}$	$\mathbf{e}'$	β
$(0, 0, 1)$	D	c_{44}	$(1, 0, 0)$	$(1, 0, 0)$	$(p_{14}^2 + p_{15}^2)/c_{44}$
$(0, 0, 1)$	D	c_{44}	$(0, 1, 0)$	$(1, 0, 0)$	$(p_{14}^2 + p_{15}^2)/c_{44}$

Table 2.4.5.16. *Cubic Laue classes C_1 and C_2: transverse modes, right-angle scattering*

This table, written for the class C_2, is also valid for the class C_1 with the additional relation $p_{12} = p_{13}$. It can also be used for the spherical system where $c_{44} = \frac{1}{2}(c_{11} - c_{12})$, $p_{44} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	β
(1, 0, 0)	D	c_{44}	(001)	(0, 0, 1)	$(1, -1, 0)/\sqrt{2}$	$p_{44}^2/2c_{44}$
(1, 0, 0)	D	c_{44}	(010)	(0, 1, 0)	$(1, 0, 1)/\sqrt{2}$	$p_{44}^2/2c_{44}$
$(1, 1, 0)/\sqrt{2}$	(0, 0, 1)	c_{44}	(001)	(0, 0, 1)	(1, 0, 0)	$p_{44}^2/2c_{44}$
$(1, 1, 0)/\sqrt{2}$	$(-1, 1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} - c_{12})$	(001)	(0, 0, 1)	(0, 0, 1)	$(p_{12} - p_{13})^2/4C$
$(1, 1, 0)/\sqrt{2}$	$(-1, 1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} - c_{12})$	(1−10)	$(1, -1, 0)/\sqrt{2}$	$(1, 1, -\sqrt{2})/2$	$(2p_{11} - p_{12} - p_{13})^2/32C$

Table 2.4.5.17. *Tetragonal T_1 and hexagonal H_1 Laue classes: transverse modes, right-angle scattering*

This table, written for the class T_1, is also valid for the class H_1 with the additional relations $c_{66} = \frac{1}{2}(c_{11} - c_{12})$; $p_{66} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	β
(1, 0, 0)	(0, 0, 1)	c_{44}	(001)	(0, 0, 1)	$(q_1^{(1)}, q_2^{(1)}, 0)$	$(q_1^{(1)}p'_{4\bar{4}})^2/c_{44}$
(1, 0, 0)	(0, 1, 0)	c_{66}	(010)	(0, 1, 0)	$(q_1^{(2)}, 0, q_3^{(2)})$	$\{[(n_3q_1^{(2)})^2 + (n_1q_3^{(2)})^2]^2/n_3^4c_{66}\}(q_1^{(2)}p_{66})^2$
(0, 0, 1)	D	c_{44}	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3q_1^{(5)})^2 + (n_1q_3^{(5)})^2]^2/n_1^4c_{44}\}(q_3^{(5)}p'_{44})^2$
$(1, 1, 0)/\sqrt{2}$	(0, 0, 1)	c_{44}	(001)	(0, 0, 1)	$(q_1^{(7)}, q_2^{(7)}, 0)$	$[(q_1^{(7)} + q_2^{(7)})p'_{4\bar{4}}]^2/2c_{44}$
$(1, 1, 0)/\sqrt{2}$	$(1, -1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} - c_{12})$	(1−10)	$(1, -1, 0)/\sqrt{2}$	$(q_1^{(10)}, q_1^{(10)}, q_3^{(10)})$	$\{[2(n_3q_1^{(10)})^2 + (n_1q_3^{(10)})^2]^2/n_3^4(c_{11} - c_{12})\}[q_1^{(10)}(p_{11} - p_{12})]^2$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66})$	(100)	(1, 0, 0)	(0, 1, 0)	$p_{66}^2/(c_{44} + c_{66})$

Table 2.4.5.18. *Hexagonal H_2 Laue class: transverse modes, right-angle scattering*

$c_{66} = \frac{1}{2}(c_{11} - c_{12})$; $p_{66} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	β
(1, 0, 0)	(0, 0, 1)	c_{44}	(001)	(0, 0, 1)	$(q_1^{(1)}, q_2^{(1)}, 0)$	$(q_1^{(1)}p'_{4\bar{4}} + q_2^{(1)}p_{45})^2/c_{44}$
(1, 0, 0)	(0, 1, 0)	c_{66}	(001)	$(1, 1, 0)/\sqrt{2}$	$(1, -1, 0)/\sqrt{2}$	p_{16}^2/c_{66}
(1, 0, 0)	(0, 1, 0)	c_{66}	(010)	(0, 1, 0)	$(q_1^{(2)}, 0, q_3^{(2)})$	$\{[(n_3q_1^{(2)})^2 + (n_1q_3^{(2)})^2]^2/n_3^4c_{66}\}(q_1^{(2)}p_{66})^2$
(0, 0, 1)	D	c_{44}	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3q_1^{(5)})^2 + (n_1q_3^{(5)})^2]^2/n_1^4c_{44}\}(q_3^{(5)})^2(p_{44}'^2 + p_{45}^2)$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66})$	(100)	(1, 0, 0)	(1, 0, 0)	$p_{16}^2/(c_{44} + c_{66})$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66})$	(100)	(1, 0, 0)	(0, 1, 0)	$p_{66}^2/(c_{44} + c_{66})$

Table 2.4.5.19. *Tetragonal T_2 Laue class: transverse modes, right-angle scattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	β
(1, 0, 0)	(0, 0, 1)	c_{44}	(001)	(0, 0, 1)	$(q_1^{(1)}, q_2^{(1)}, 0)$	$(q_1^{(1)}p'_{4\bar{4}} + q_2^{(1)}p_{45})^2/c_{44}$
(1, 0, 0)	(0, 0, 1)	c_{44}	(010)	(0, 1, 0)	$(q_1^{(2)}, 0, q_3^{(2)})$	$\{[(n_3q_1^{(2)})^2 + (n_1q_3^{(2)})^2]^2/n_1^4c_{44}\}(q_3^{(2)}p_{45})^2$
(0, 0, 1)	D	c_{44}	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3q_1^{(5)})^2 + (n_1q_3^{(5)})^2]^2/n_1^4c_{44}\}(q_3^{(5)})^2(p_{44}'^2 + p_{45}^2)$
$(1, 1, 0)/\sqrt{2}$	(0, 0, 1)	c_{44}	(001)	(0, 0, 1)	$(q_1^{(7)}, q_2^{(7)}, 0)$	$[(q_1^{(7)} + q_2^{(7)})p'_{4\bar{4}} + (q_2^{(7)} - q_1^{(7)})p_{45}]^2/2c_{44}$

Table 2.4.5.20. *Orthorhombic Laue class O: transverse modes, right-angle scattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	β
(1, 0, 0)	(0, 0, 1)	c_{55}	(001)	(0, 0, 1)	$(q_1^{(1)}, q_2^{(1)}, 0)$	$\{[(n_2q_1^{(1)})^2 + (n_1q_2^{(1)})^2]^2/n_2^4c_{55}\}(q_1^{(1)}p'_{55})^2$
(1, 0, 0)	(0, 1, 0)	c_{66}	(010)	(0, 1, 0)	$(q_1^{(2)}, 0, q_3^{(2)})$	$\{[(n_3q_1^{(2)})^2 + (n_1q_3^{(2)})^2]^2/n_3^4c_{66}\}(q_1^{(2)}p'_{6\bar{6}})^2$
(0, 1, 0)	(1, 0, 0)	c_{66}	(100)	(1, 0, 0)	$(0, q_2^{(3)}, q_3^{(3)})$	$\{[(n_3q_2^{(3)})^2 + (n_2q_3^{(3)})^2]^2/n_3^4c_{66}\}(q_2^{(3)}p'_{66})^2$
(0, 1, 0)	(0, 0, 1)	c_{44}	(001)	(0, 0, 1)	$(q_1^{(4)}, q_2^{(4)}, 0)$	$\{[(n_2q_1^{(4)})^2 + (n_1q_2^{(4)})^2]^2/n_1^4c_{44}\}(q_2^{(4)}p'_{4\bar{4}})^2$
(0, 0, 1)	(0, 1, 0)	c_{44}	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3q_1^{(5)})^2 + (n_1q_3^{(5)})^2]^2/n_1^4c_{44}\}(q_3^{(5)}p'_{44})^2$
(0, 0, 1)	(1, 0, 0)	c_{55}	(100)	(1, 0, 0)	$(0, q_2^{(6)}, q_3^{(6)})$	$\{[(n_3q_2^{(6)})^2 + (n_2q_3^{(6)})^2]^2/n_2^4c_{55}\}(q_3^{(6)}p'_{5\bar{5}})^2$
$(1, 1, 0)/\sqrt{2}$	(0, 0, 1)	$\frac{1}{2}(c_{44} + c_{55})$	(001)	(0, 0, 1)	$(q_1^{(7)}, q_2^{(7)}, 0)$	$\{[(n_2q_1^{(7)})^2 + (n_1q_2^{(7)})^2]^2/n_1^4n_2^4(c_{44} + c_{55})\}$ $\times(n_1^2q_1^{(7)}p'_{55} + n_2^2q_2^{(7)}p'_{4\bar{4}})^2$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{55} + c_{66})$	(100)	(1, 0, 0)	$(0, q_2^{(8)}, q_3^{(8)})$	$\{[(n_3q_2^{(8)})^2 + (n_2q_3^{(8)})^2]^2/n_2^4n_3^4(c_{55} + c_{66})\}$ $\times(n_2^2q_2^{(8)}p'_{66} + n_3^2q_3^{(8)}p'_{5\bar{5}})^2$
$(1, 0, 1)/\sqrt{2}$	(0, 1, 0)	$\frac{1}{2}(c_{44} + c_{66})$	(010)	(0, 1, 0)	$(q_1^{(9)}, 0, q_3^{(9)})$	$\{[(n_1q_3^{(9)})^2 + (n_3q_1^{(9)})^2]^2/n_1^4n_3^4(c_{44} + c_{66})\}$ $\times(n_3^2q_3^{(9)}p'_{44} + n_1^2q_1^{(9)}p'_{6\bar{6}})^2$

Table 2.4.5.21. *Trigonal Laue class R_1: transverse modes, right-angle scattering*

$c_{66} = \frac{1}{2}(c_{11} - c_{12})$; $p_{66} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	β
(0, 1, 0)	(1, 0, 0)	c_{66}	(100)	(1, 0, 0)	$(0, q_2^{(3)}, q_3^{(3)})$	$\{[(n_3q_2^{(3)})^2 + (n_1q_3^{(3)})^2]^2/n_1^4n_3^4c_{66}\}(n_1^2q_2^{(3)}p_{66} + n_3^2q_3^{(3)}p_{41})^2$
(0, 1, 0)	(1, 0, 0)	c_{66}	(001)	(0, 0, 1)	$(q_1^{(4)}, q_2^{(4)}, 0)$	$(q_1^{(4)}p_{41})^2/c_{66}$
(0, 0, 1)	D	c_{44}	(010)	(0, 1, 0)	(0, 1, 0)	p_{14}^2/c_{44}
(0, 0, 1)	D	c_{44}	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3q_1^{(5)})^2 + (n_1q_3^{(5)})^2]^2/n_1^4n_3^4c_{44}\}[n_1^4(q_1^{(5)}p_{14})^2 + n_3^4(q_3^{(5)}p'_{44})^2]$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66}) + c_{14}$	(100)	(1, 0, 0)	(0, 1, 0)	$(p_{66} + p_{14})^2/2C$
$(0, -1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66}) - c_{14}$	(100)	(1, 0, 0)	(0, 1, 0)	$(p_{66} - p_{14})^2/2C$

Table 2.4.5.22. *Trigonal Laue class R_2: transverse modes, right-angle scattering*

$c_{66} = \frac{1}{2}(c_{11} - c_{12})$; $p_{66} = \frac{1}{2}(p_{11} - p_{12})$.

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	C	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	β
(0, 0, 1)	D	c_{44}	(010)	(0, 1, 0)	(0, 1, 0)	p_{14}^2/c_{44}
(0, 0, 1)	D	c_{44}	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3q_1^{(5)})^2 + (n_1q_3^{(5)})^2]^2/n_1^4n_3^4c_{44}\}[n_1^4(q_1^{(5)}p_{14})^2 + n_3^4(q_3^{(5)}p'_{44})^2]$

Table 2.4.5.23. *Particular directions of incident light used in Tables 2.4.5.17 to 2.4.5.22*

$\varepsilon_1 = (n_2 + n_3 - 2n_1)/4n_1$, $\varepsilon_2 = (n_1 + n_3 - 2n_2)/4n_2$, $\varepsilon_3 = (n_1 + n_2 - 2n_3)/4n_3$.

Notation	q_1	q_2	q_3
$\mathbf{q}^{(1)}$	$-2^{(-1/2)}(1 - \varepsilon_3)$	$2^{(-1/2)}(1 + \varepsilon_3)$	0
$\mathbf{q}^{(2)}$	$-2^{(-1/2)}(1 - \varepsilon_2)$	0	$2^{(-1/2)}(1 + \varepsilon_2)$
$\mathbf{q}^{(3)}$	0	$-2^{(-1/2)}(1 - \varepsilon_1)$	$2^{(-1/2)}(1 + \varepsilon_1)$
$\mathbf{q}^{(4)}$	$2^{(-1/2)}(1 + \varepsilon_3)$	$-2^{(-1/2)}(1 - \varepsilon_3)$	0
$\mathbf{q}^{(5)}$	$2^{(-1/2)}(1 + \varepsilon_2)$	0	$-2^{(-1/2)}(1 + \varepsilon_2)$
$\mathbf{q}^{(6)}$	0	$2^{(-1/2)}(1 + \varepsilon_1)$	$-2^{(-1/2)}(1 - \varepsilon_1)$
$\mathbf{q}^{(7)}$	$-2^{(-1/2)}(n_1 + n_3)(n_1^2 + n_3^2)^{(-1/2)}$	$2^{(-1/2)}(n_1 - n_3)(n_1^2 + n_3^2)^{(-1/2)}$	0
$\mathbf{q}^{(8)}$	0	$-2^{(-1/2)}(n_1 + n_2)(n_1^2 + n_2^2)^{(-1/2)}$	$2^{(-1/2)}(n_2 - n_1)(n_1^2 + n_2^2)^{(-1/2)}$
$\mathbf{q}^{(9)}$	$2^{(-1/2)}(n_3 - n_2)(n_2^2 + n_3^2)^{(-1/2)}$	0	$-2^{(-1/2)}(n_2 + n_3)(n_2^2 + n_3^2)^{(-1/2)}$
$\mathbf{q}^{(10)}$	$-\frac{1}{2}(1 - \varepsilon_2)$	$-\frac{1}{2}(1 - \varepsilon_2)$	$2^{(-1/2)}(1 + \varepsilon_2)$

optical setup, the collection and acceptance angles of the instruments, spurious reflections and spurious interferences, *etc*. A full list is too long to be given here. However, when properly executed, interferometry is a fine tool, the performance of which is unequalled in its frequency range.

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