Remark. It is possible to construct multiple twins that cannot be treated as a cyclic sequence of binary twin elements. This case occurs if a pair of domain states 1 and 2 are related only by an n-fold rotation or roto-inversion ($n \geq 3$). The resulting coset again contains the alternative twin operations, but in this case *only* for the orientation relation $1 \Rightarrow 2$, and not for $2 \Rightarrow 1$ ('non-transposable' domain pair). This coset procedure thus does not result in a composite group for a domain pair. In order to obtain the composite group, further cosets have to be constructed by means of the higher powers of the twin rotation under consideration. Each new power corresponds to a further domain state and twin law.

This construction leads to a composite symmetry $\mathcal{K}(n)$ of supergroup index $[i] \geq 3$ with respect to the *eigensymmetry* $\mathcal{H}$. This case can occur only for the following $\mathcal{H} \Rightarrow \mathcal{K}$ pairs: $1 \Rightarrow 3$, $\bar{1} \Rightarrow \bar{3}$, $1 \Rightarrow 4$, $1 \Rightarrow \bar{4}$, $m \Rightarrow 4/m$, $1 \Rightarrow 6$, $2 \Rightarrow 6$, $m \Rightarrow \bar{6} = 3/m$, $m \Rightarrow 6/m$, $2/m \Rightarrow 6/m$ (monoaxial point groups), as well as for the two cubic pairs $222 \Rightarrow 23$, $mmm \Rightarrow 2/m\bar{3}$. For the pairs $1 \Rightarrow 3$, $\bar{1} \Rightarrow \bar{3}$, $m \Rightarrow \bar{6} = 3/m$, $2/m \Rightarrow 6/m$ and the two cubic pairs $222 \Rightarrow 23$, $mmm \Rightarrow 2/m\bar{3}$, the $\mathcal{K}$ relations are of index [3] and imply three non-transposable domain states. For the pairs $1 \Rightarrow 4$, $1 \Rightarrow \bar{4}$, $m \Rightarrow 4/m$, as well as $1 \Rightarrow 6$ and $m \Rightarrow 6/m$, four or six different domain states occur. Among them, however, domain pairs related by the second powers of 4 and $\bar{4}$ as well as by the third powers of 6 and $\bar{6}$ operations are transposable, because these twin operations correspond to twofold rotations or, for $\bar{6}$, to m.

No growth twins of this type are known so far. As a transformation twin, langbeinite ($23 \Longleftrightarrow 222$) is the only known example.

3.3.5. Description of the twin law by black–white symmetry

An alternative description of twinning employs the symbolism of colour symmetry. This method was introduced by Curien & Le Corre (1958) and by Curien & Donnay (1959). In this approach, a colour is attributed to each different domain state. Depending on the number of domain states, simple twins with two colours (*i.e.* 'black–white' or 'dichromatic' or 'anti-symmetry' groups) and multiple twins with more than two colours (*i.e.* 'polychromatic' symmetry groups) have to be considered. Two kinds of operations are distinguished:

(i) The symmetry operations of the *eigensymmetry* (point group) of the crystal. These operations are 'colour-preserving' and form the 'monochromatic' *eigensymmetry* group $\mathcal{H}$. The symbols of these operations are unprimed.

(ii) The twin operations, *i.e.* those operations which transform one orientation state into another, are 'colour-changing' operations. Their symbols are designated by a prime if of order 2: $2'$, m', $\bar{1}'$.

For *simple twins*, all colour-changing (twin) operations are binary, hence the two domain states are transposable. The composite symmetry $\mathcal{K}$ of these twins thus can be described by a 'black-and-white' symmetry group. The coset, which defines the twin law, contains only colour-changing (primed) operations. This notation has been used already in previous sections.

It should be noted that symbols such as $4'$ and $6'$, despite appearance to the contrary, represent *binary* black-and-white operations, because $4'$ contains 2, and $6'$ contains 3 and $2'$, with $2'$ being the twin operation. For this reason, these symbols are written here as $4'(2)$ and $6'(3)$, whereby the unprimed symbol in parentheses refers to the *eigensymmetry* part of the twin axis. In contrast, $6'(2)$ would designate a (polychromatic) twin axis which relates three domain states (three colours), each of *eigensymmetry* 2. Twin centres of symmetry $\bar{1}'$ are always added to the symbol in order to bring out an inversion twinning contained in the twin law. In the original version of Curien & Donnay (1959), the black–white symbols were only used for twinning by merohedry. In the present chapter, the symbols are also applied to non-merohedral twins, as is customary for (ferroelastic) domain structures. This has the consequence, however, that the *eigensymmetries* $\mathcal{H}$ or $\mathcal{H}^*$ and the composite symmetries $\mathcal{K}$ or $\mathcal{K}^*$ may belong to different crystal systems and, thus, are referred to different coordinate systems, as shown for the composite symmetry of gypsum in Section 3.3.4.1.

For the treatment of *multiple twins*, 'polychromatic' composite groups $\mathcal{K}(n)$ are required. These contain colour-changing operations of order higher than 2, *i.e.* they relate three or more colours (domain states). Consequently, not all pairs of domain states are transposable. This treatment of multiple twins is rather complicated and only sensible if the composite symmetry group is finite and contains twin axes of low order ($n \leq 8$). For this reason, the symbols for the composite symmetry $\mathcal{K}$ of multiple twins are written without primes; see the examples in Section 3.3.4.4(iii). An extension of the dichromatic twin descriptions to polychromatic symbols for multiple twins was recently presented by Nespolo (2004).

3.3.6. Examples of twinned crystals

In order to illustrate the foregoing rather abstract deliberations, an extensive set of examples of twins occurring either in nature or in the laboratory is presented below. In each case, the twin law is described in two ways: by the coset of alternative twin operations and by the black–white symmetry symbol of the composite symmetry $\mathcal{K}$, as described in Sections 3.3.4 and 3.3.5.

For the description of a twin, the conventional crystallographic coordinate system of the crystal and its *eigensymmetry* group $\mathcal{H}$ are used in general; exceptions are specifically stated. To indicate the orientation of the twin elements (both rational and irrational) and the composition planes, no specific convention has been adopted; rather a variety of intuitively understandable simple symbols are chosen for each particular case, with the additional remark 'rational' or 'irrational' where necessary. Thus, for twin reflection planes and (planar) twin boundaries symbols such as m_x, $m(100)$, $m \parallel (100)$ or $m \perp [100]$ are used, whereas twin rotation axes are designated by 2_z, $2_{[001]}$, $2 \parallel [001]$, $2 \perp (001)$, 3_z, $3_{[111]}$, $4_{[001]}$ *etc.*

3.3.6.1. *Macroscopic identification of twins and of twin laws*

As an introduction to the subsequent examples, this section shows how to recognize and identify twinning in a crystal, either by morphological features and observations in polarized light, or by etching, decoration and X-ray diffraction topography. Diffraction effects of twins are treated in Section 3.3.11.

(i) *Re-entrant angles and twin striations*

The most prominent and easily recognizable morphological features are exhibited by penetration and contact twins with their *re-entrant angles* (*edges*). Re-entrant edges are typical for twins with non-parallel lattices (non-merohedral twins) and reticular merohedral twins with a $\Sigma > 1$ coincidence lattice (*e.g.* $\Sigma 3$ spinel twins). Merohedral $\Sigma 1$ twins with full lattice parallelism in general do not exhibit re-entrant edges but can often be identified by twin-related faces appearing in the morphology of a twinned crystal (*e.g.* $\Sigma 1$ twins of quartz). The re-entrant edges mark the outcrop of the twin boundaries, which are defined by the path of this edge during crystal growth. Note that re-entrant edges occur in any arbitrary intergrowth of crystals, which can sometimes be misinterpreted as twinning.

Illustrations of penetration twins are Figs. 3.3.6.6 ($FeBO_3$), 3.3.6.8 (spinel law, diamond), 3.3.6.15 (staurolite) and 3.3.7.1 (orthoclase feldspar). Contact growth twins with re-entrant angles are presented in Fig. 3.3.6.3 (gypsum) and 3.3.6.7 (calcite). In repeated growth twins (*e.g.* albite, Fig. 3.3.6.13) the parallel re-entrant edges form 'polysynthetic twins'. If the width of the twin lamellae gets smaller and smaller, *twin striations* develop as a characteristic feature of these twins. For further examples see the

