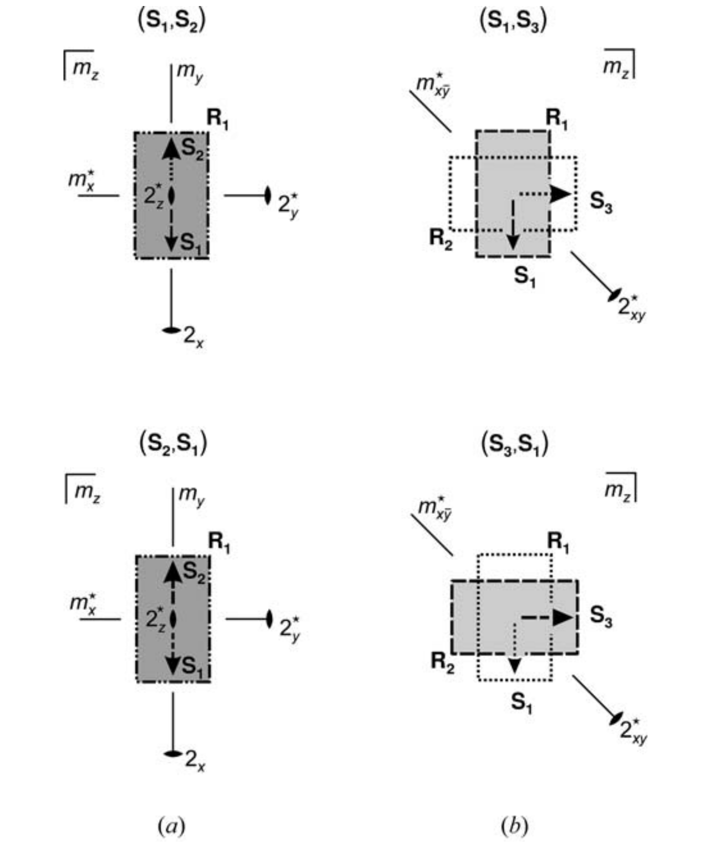

Fig. 3.4.3.1. Transposable domain pairs. Single-domain states are those from Fig. 3.4.2.2. (*a*) Completely transposable non-ferroelastic domain pair. (*b*) Partially transposable ferroelastic domain pair.

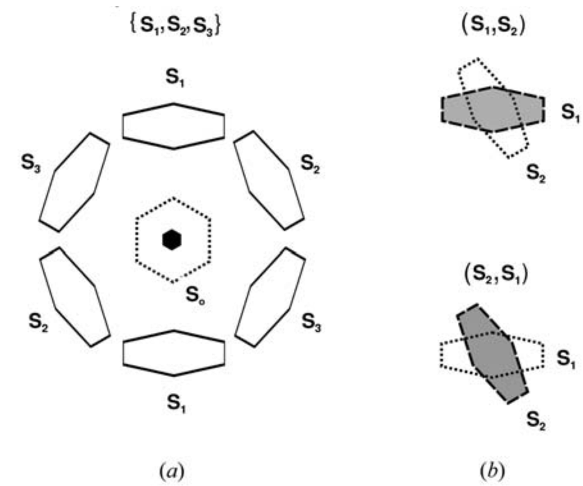

Fig. 3.4.3.2. Non-transposable domain pairs. (*a*) The parent phase with symmetry $G = 6_z/m_z$ is represented by a dotted hexagon and the three ferroelastic single-domain states with symmetry $F_1 = F_2 = F_3 = 2_z/m_z$ are depicted as drastically squeezed hexagons. (*b*) Domain pair $(\mathbf{S}_1, \mathbf{S}_2)$ and transposed domain pair $(\mathbf{S}_2, \mathbf{S}_1)$. There exists no operation from the group $6_z/m_z$ that would exchange domain states $\mathbf{S}_1$ and $\mathbf{S}_2$, *i.e.* that would transform one domain pair into a transposed domain pair.

Restriction (ii), formulated by Georges Friedel (1926) and explained in detail by Cahn (1954), expresses a necessity to exclude from considerations crystal aggregates (intergrowths) with approximate or accidental 'nearly exact' crystal components resembling twins (Friedel's *macles d'imagination*) and thus to restrict the definition to 'true twins' that fulfil condition (i) exactly and are characteristic for a given material. If we confine our considerations to domain structures that are formed from a *homogeneous* parent phase, this requirement is fulfilled for *all* aggregates consisting of two or more domains. Then the definition of a 'domain twin law' is expressed only by condition (i). Condition (ii) is important for growth twins.

We should note that the definition of a twin law given above involves only domain states and does not explicitly contain specification of the contact region between twin components or neighbouring domains. The concept of domain state is, therefore, relevant for discussing the twin laws. Moreover, there is no requirement on the coexistence of interpenetrating structures in a domain pair. One can even, therefore, consider cases where no real coexistence of both structures is possible. Nevertheless, we note that the characterization of twin laws used in mineralogy often includes specification of the contact region (*e.g.* twin plane or diffuse region in penetrating twins).

Ordered domain pairs $(\mathbf{S}_1, \mathbf{S}_2)$ and $(\mathbf{S}_1, \mathbf{S}_3)$, formed from domain states of our illustrative example (see Fig. 3.4.2.2), are displayed in Fig. 3.4.3.1(*a*) and (*b*), respectively, as two superposed rectangles with arrows representing spontaneous polarization. In ordered domain pairs, the first and the second domain state are distinguished by shading [the first domain state is grey ('black') and the second clear ('white')] and/or by using dashed and dotted lines for the first and second domain state, respectively.

In Fig. 3.4.3.2, the ordered domain pair $(\mathbf{S}_1, \mathbf{S}_2)$ and the transposed domain pair $(\mathbf{S}_2, \mathbf{S}_1)$ are depicted in a similar way for another example with symmetry descent $G = 6_z/m_z \supset 2_z/m_z = F_1$.

Let us now examine the *symmetry of domain pairs*. The *symmetry group* F_{ik} *of an ordered domain pair* $(\mathbf{S}_i, \mathbf{S}_k) = (\mathbf{S}_i, g_{ik}\mathbf{S}_i)$ consists of all operations that leave invariant both $\mathbf{S}_i$ and $\mathbf{S}_k$, *i.e.* F_{ik} comprises all operations that are common to stabilizers (symmetry groups) F_i and F_k of domain states $\mathbf{S}_i$ and $\mathbf{S}_k$, respectively,

$$F_{ik} \equiv F_i \cap F_k = F_i \cap g_{ik} F_i g_{ik}^{-1}, \qquad (3.4.3.8)$$

where the symbol $\cap$ denotes the intersection of groups F_i and F_k. The group F_{ik} is in Section 3.3.4 denoted by $\mathcal{H}^*$ and is called an intersection group.

From equation (3.4.3.8), it immediately follows that the symmetry F_{ki} of the transposed domain pair $(\mathbf{S}_k, \mathbf{S}_i)$ is the same as the symmetry F_{ik} of the initial domain pair $(\mathbf{S}_i, \mathbf{S}_k)$:

$$F_{ki} = F_k \cap F_i = F_i \cap F_k = F_{ik}. \qquad (3.4.3.9)$$

Symmetry operations of an unordered domain pair $\{\mathbf{S}_i, \mathbf{S}_k\}$ include, besides operations of F_{ik} that do not change either $\mathbf{S}_i$ or $\mathbf{S}_k$, all transposing operations, since for an unordered domain pair a transposed domain pair is identical with the initial domain pair [see equation (3.4.3.4)]. If $g^\star_{ik}$ is a transposing operation of $(\mathbf{S}_i, \mathbf{S}_k)$, then all operations from the left coset $g^\star_{ik} F_{ik}$ are transposing operations of that domain pair as well. Thus the *symmetry group* J_{ik} *of an unordered domain pair* $\{\mathbf{S}_i, \mathbf{S}_k\}$ can be, in a general case, expressed in the following way:

$$J_{ik} = F_{ik} \cup g^\star_{ik} F_{ik}, \quad g^\star_{ik} \in G. \qquad (3.4.3.10)$$

Since, for an unordered domain, the order of domain states in a domain pair is not significant, the transposition of indices i, k in J_{ik} does not change this group,

$$J_{ik} = F_{ik} \cup g^\star_{ik} F_{ik} = F_{ki} \cup g^\star_{ki} F_{ki} = J_{ki}, \qquad (3.4.3.11)$$

which also follows from equations (3.4.3.3) and (3.4.3.9).

A *basic classification of domain pairs* follows from their symmetry. Domain pairs for which at least one transposing operation exists are called *transposable* (or *ambivalent*) *domain pairs*. The symmetry group of a transposable unordered domain pair $(\mathbf{S}_i, \mathbf{S}_k)$ is given by equation (3.4.3.10).