International Tables for Crystallography (2013). Vol. D, Figure 3.4.3.3, p. 515.

$a \neq 0$, this domain pair can be classified as a *ferroelectric domain pair*.

Similarly, the first number a in column g_μ determines the number of independent components of the tensor of optical activity that have opposite sign in domain states $\mathbf{S}_1$ and $\mathbf{S}_j$; if $a \neq 0$, the two domain states in the pair can be distinguished by optical activity. Such a domain pair can be called a *gyrotropic domain pair*. As in Table 3.4.3.1 for the ferroelectric (ferroelastic) domain pairs, we can define a *gyrotropic phase* as a ferroic phase with gyrotropic domain pairs. The corresponding phase transition to a gyrotropic phase is called a *gyrotropic phase transition* (Koňák *et al.*, 1978; Wadhawan, 2000). If it is possible to switch gyrotropic domain states by an external field, the phase is called a *ferrogyrotropic phase* (Wadhawan, 2000). Further division into full and partial subclasses is possible.

One can also define *piezoelectric (electro-optic) domain pairs*, *electrostrictive (elasto-optic) domain pairs* and corresponding phases and transitions.

As we have already stated, domain states in a domain pair $(\mathbf{S}_1, \mathbf{S}_j)$ differ in principal tensor parameters of the transition $K_{1j} \supset F_1$. These principal tensor parameters are Cartesian tensor components or their linear combinations that transform according to an irreducible representation Γ_α specifying the primary order parameter of the transition $K_{1j} \supset F_1$ (see Section 3.1.3). Owing to a special form of K_{1j} expressed by equation (3.4.3.42), this representation is a real one-dimensional irreducible representation of K_{1j}. Such a representation associates +1 with operations of F_1 and -1 with operations from the left coset $g_{1j}^\star$. This means that the principal tensor parameters are one-dimensional and have the same absolute value but opposite sign in $\mathbf{S}_1$ and $\mathbf{S}_j = g_{1j}^\star \mathbf{S}_1$. Principal tensor parameters for symmetry descents $K_{1j} \supset F_1$ and associated Γ_α's of all non-ferroelastic domain pairs can be found for property tensors of lower rank in Table 3.1.3.1 and for all tensors appearing in Table 3.4.3.5 in the software *GI★KoBo*-1 and in Kopský (2001).

These specific properties of non-ferroelastic domain pairs allow one to formulate simple rules for tensor distinction that do not use principal tensor parameters and that are applicable for property tensors of lower rank.

(i) Symmetry descents $K_{1j} \supset F_1$ of non-ferroelastic domain pairs for lower-rank property tensors lead only to the appearance of independent Cartesian morphic tensor components and not to the breaking of relations between these components. These morphic Cartesian tensor components can be found by comparing matrices of property tensors in the twinning group K_{1j} and the low-symmetry group F_1 as those components that appear in F_1 but are zero in K_{1j}.

(ii) As follows from Table 3.4.3.4, one can always find a twinning operation that is either inversion, or a twofold axis or a mirror plane with a prominent crystallographic orientation. By applying the method of direct inspection (see Section 1.1.4.6.3), one can in most cases easily find morphic Cartesian components in the second domain state of the domain pair considered and prove that they differ only in sign.

Example 3.4.3.4. Tensor distinction of domains and switching in lead germanate. Lead germanate ($Pb_5Ge_3O_{11}$) undergoes a phase transition with symmetry descent $G = \bar{6} \supset 3 = F_1$ for which we find in Table 3.4.2.7, column K_{1j}, just one twinning group $K_{1j} = \bar{6}^\star$, *i.e.* $K_{1j}^\star = G$. This means that there is only one G-orbit of domain pairs. Since Fam3 = Fam$\bar{6}$ [see Table 3.4.2.2 and equation (3.4.3.40)] this orbit comprises non-ferroelastic domain pairs. In Table 3.4.3.4, we find for $F_1 = 3$ and $F_{1j}^\star = \bar{6}$ that the two domain states differ in some components of all property tensors listed in this table. The first polar tensor is the spontaneous polarization (the pair is ferroelectric) with one component ($a = 1$) that has opposite sign in the two domain states. In Table 3.1.3.1, we find for $G(= K_{1j}) = \bar{6}$ and $F_1 = 3$ that this component is $P_3 = P_z$. From Table 3.4.3.1, it follows that the state shift is electrically first order with switching field $\mathbf{E} = (0, 0, E_z)$.

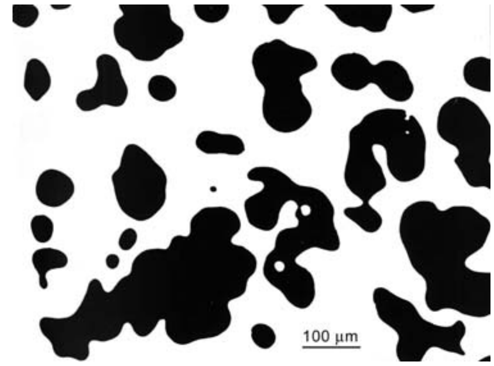

Fig. 3.4.3.3. Domain structure in lead germanate observed using a polarized-light microscope. Visualization based on the opposite sign of the optical activity coefficient in the two domain states. Courtesy of Vl. Shur, Ural State University, Ekaterinburg.

The first optical tensor, which could enable the visualization of the domain states, is the optical activity g_μ with two independent components which have opposite sign in the two domain states. In the software *GI★KoBo*-1, path: *Subgroups\View\Domains* or in Kopský (2001) we find these components: $g_3, g_1 + g_2$. Shur *et al.* (1989) have visualized in this way the domain structure of lead germanate with excellent black and white contrast (see Fig. 3.4.3.3). Other examples are given in Shuvalov & Ivanov (1964) and especially in Koňák *et al.* (1978).

Table 3.4.3.4 can be used readily for twinning by merohedry [see Chapter 3.3 and *e.g.* Cahn (1954); Koch (2004)], where it enables an easy determination of the tensor distinction of twin components and the specification of external fields for possible switching and detwinning.

Example 3.4.3.5. Tensor distinction and switching of Dauphiné twins in quartz. Quartz undergoes a phase transition from $G = 6_z2_x2_y$ to $F_1 = 3_z2_x$. Using the same procedure as in the previous example, we come to following conclusions: There are only two domain states $\mathbf{S}_1, \mathbf{S}_2$ and the twinning group, expressing the twin law, is equal to the high-symmetry group $K_{12}^\star = 6_z2_x2_z$. In Table 3.4.3.4, we find that these two states differ in one independent component of the piezoelectric tensor and in one elastic compliance component. Comparison of the matrices for $6_z2_x2_y$ and 3_z2_x (see Sections 1.1.4.10.3 and 1.1.4.10.4) yields the following morphic tensor components in the first domain state $\mathbf{S}_1$: $d_{11}^{(1)} = -d_{12}^{(1)} = -2d_{26}^{(1)}$ and $s_{14}^{(1)} = -s_{24}^{(1)} = 2s_{56}^{(1)}$. According to the rule given above, the values of morphic components in the second domain state $\mathbf{S}_2$ are $d_{11}^{(2)} = -d_{11}^{(1)} = -d_{12}^{(2)} = d_{12}^{(1)} = -2d_{26}^{(2)} = 2d_{26}^{(1)}$ and $s_{14}^{(2)} = -s_{14}^{(1)} = -s_{24}^{(2)} = s_{24}^{(1)} = 2s_{56}^{(2)} = -2s_{56}^{(1)}$ [see Section 3.4.5 (Glossary)]. These results show that there is an elastic state shift of second order and an electromechanical state shift of second order. Nonzero components $d_{14} = -d_{25}$ in $6_z2_x2_y$ are the same in both domain states. Similarly, one can find five independent components of the tensor $s_{\mu\nu}$ that are nonzero in $6_z2_x2_y$ and equal in both domain states. For the piezo-optic tensor $\pi_{\mu\nu}$, one can proceed in a similar way. Aizu (1973) has used the ferrobielastic character of the domain pairs for visualizing domains and realizing switching in quartz. Other methods for switching and visualizing domains in quartz are known (see *e.g.* Bertagnolli *et al.*, 1978, 1979).

3.4.3.6. *Ferroelastic domain pairs*

A *ferroelastic domain pair* consists of two domain states that have different spontaneous strain. A domain pair $(\mathbf{S}_1, \mathbf{S}_j)$ is a ferroelastic domain pair if the crystal family of its twinning group

references