

## 3. SYMMETRY ASPECTS OF PHASE TRANSITIONS, TWINNING AND DOMAIN STRUCTURES

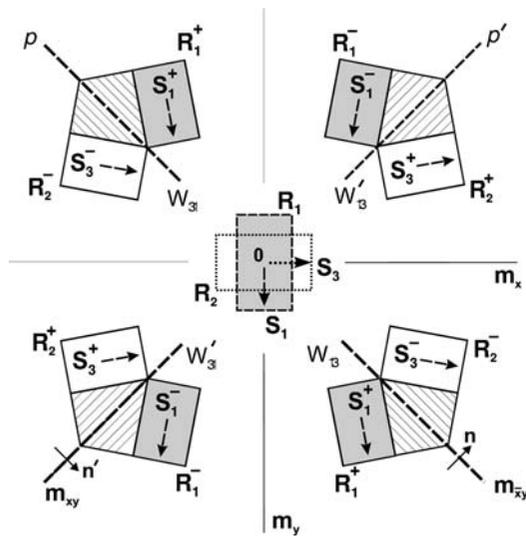


Fig. 3.4.3.8. Exploded view of four ferroelastic twins with disoriented ferroelastic domain states  $\mathbf{R}_1^+$ ,  $\mathbf{R}_2^-$  and  $\mathbf{R}_1^-$ ,  $\mathbf{R}_2^+$  formed from a single-domain pair ( $\mathbf{S}_1$ ,  $\mathbf{S}_2$ ) (in the centre).

and positive sides of  $\mathbf{n}$ , respectively, for which we have used letters  $Q_1$  and  $Q_2$ , respectively. Then  $(\mathbf{R}_1^+ | \mathbf{R}_2^-)$  represent domains  $\mathbf{D}_1(\mathbf{R}_1^+, Q_1)$  and  $\mathbf{D}_2(\mathbf{R}_2^-, Q_2)$ , respectively. The symbol  $(\mathbf{R}_1^+ | \mathbf{R}_2^-)$  properly specifies a domain twin with a zero-thickness domain wall.

A domain wall can be considered as a domain twin with domain regions restricted to non-homogeneous parts near the plane  $p$ . For a domain wall in domain twin  $(\mathbf{R}_1^+ | \mathbf{R}_2^-)$  we shall use the symbol  $[\mathbf{R}_1^+ | \mathbf{R}_2^-]$ , which expresses the fact that a domain wall of zero thickness needs the same specification as the domain twin.

If we exchange domain states in the twin  $(\mathbf{R}_1^+ | \mathbf{n} | \mathbf{R}_2^-)$ , we get a *reversed twin (wall)* with the symbol  $(\mathbf{R}_2^- | \mathbf{n} | \mathbf{R}_1^+)$ . These two ferroelastic twins are depicted in the lower right and upper left parts of Fig. 3.4.3.8, where – for ferroelastic–non-ferroelectric twins – we neglect spontaneous polarization of ferroelastic domain states. The reversed twin  $\mathbf{R}_2^- | \mathbf{n} | \mathbf{R}_1^+$  has the opposite shear direction.

Twin and reversed twin can be, but may not be, crystallographically equivalent. Thus *e.g.* ferroelastic–non-ferroelectric twins  $(\mathbf{R}_1^+ | \mathbf{n} | \mathbf{R}_2^-)$  and  $(\mathbf{R}_2^- | \mathbf{n} | \mathbf{R}_1^+)$  in Fig. 3.4.3.8 are equivalent, *e.g. via*  $2_z$ , whereas ferroelastic–ferroelectric twins  $(\mathbf{S}_1^+ | \mathbf{n} | \mathbf{S}_3^-)$  and  $(\mathbf{S}_3^- | \mathbf{n} | \mathbf{S}_1^+)$  are not equivalent, since there is no operation in the group  $K_{12}$  that would transform  $(\mathbf{S}_1^+ | \mathbf{n} | \mathbf{S}_3^-)$  into  $(\mathbf{S}_3^- | \mathbf{n} | \mathbf{S}_1^+)$ .

As we shall show in the next section, the symmetry group  $T_{12}(\mathbf{n})$  of a twin and the symmetry group  $T_{21}(\mathbf{n})$  of a reverse twin are equal,

$$T_{12}(\mathbf{n}) = T_{21}(\mathbf{n}). \quad (3.4.3.66)$$

A sequence of repeating twins and reversed twins

$$\dots \mathbf{R}_1^+ | \mathbf{n} | \mathbf{R}_2^- | \mathbf{n} | \mathbf{R}_1^+ | \mathbf{n} | \mathbf{R}_2^- | \mathbf{n} | \mathbf{R}_1^+ | \mathbf{n} | \mathbf{R}_2^- | \mathbf{n} | \mathbf{R}_1^+ | \mathbf{n} | \mathbf{R}_2^- \dots \quad (3.4.3.67)$$

forms a *lamellar ferroelastic domain structure* that is very common in ferroelastic phases (see *e.g.* Figs. 3.4.1.1 and 3.4.1.4).

Similar considerations can be applied to the second equally deformed plane  $p'$  that is perpendicular to  $p$ . The two twins and corresponding compatible domain walls for the equally deformed plane  $p'$  have the symbols  $(\mathbf{R}_1^- | \mathbf{n}' | \mathbf{R}_2^+)$  and  $(\mathbf{R}_2^+ | \mathbf{n}' | \mathbf{R}_1^-)$ , and are also depicted in Fig. 3.4.3.8. The corresponding lamellar domain structure is

$$\dots \mathbf{R}_1^- | \mathbf{n}' | \mathbf{R}_2^+ | \mathbf{n}' | \mathbf{R}_1^- | \mathbf{n}' | \mathbf{R}_2^+ | \mathbf{n}' | \mathbf{R}_1^- | \mathbf{n}' | \mathbf{R}_2^+ | \mathbf{n}' | \mathbf{R}_1^- | \mathbf{n}' | \mathbf{R}_2^+ \dots \quad (3.4.3.68)$$

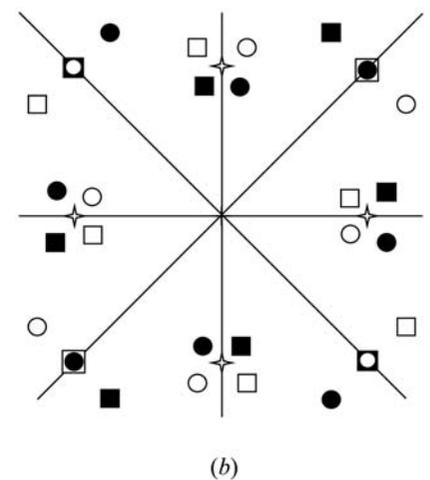
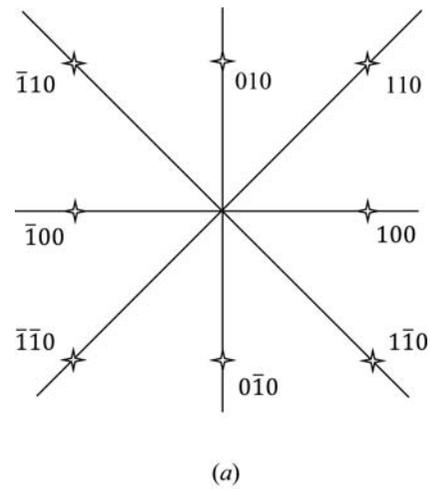


Fig. 3.4.3.9. Splitting of diffraction spots at a tetragonal-to-orthorhombic transition. (a) Fragment of the reciprocal lattice (stars) of the parent tetragonal phase. (b) Superposition of the reciprocal lattices of the four ferroelastic domains states in Fig. 3.4.3.8:  $\mathbf{R}_1^+$  (reciprocal lattice black squares),  $\mathbf{R}_1^-$  (reciprocal lattice black circles),  $\mathbf{R}_2^+$  (reciprocal lattice white squares) and  $\mathbf{R}_2^-$  (reciprocal lattice white circles).

Thus from one ferroelastic single-domain pair  $(\mathbf{R}_1, \mathbf{R}_2)$  depicted in the centre of Fig. 3.4.3.8 four different ferroelastic domain twins can be formed. It can be shown that these four twins have the same shear angle  $\omega$  and the same amount of shear  $s$ . They differ only in the direction of the shear.

Four disoriented domain states  $\mathbf{R}_1^-, \mathbf{R}_1^+$  and  $\mathbf{R}_2^-, \mathbf{R}_2^+$  that appear in the four domain twins considered above are related by lost operations (*e.g.* diagonal, vertical and horizontal reflections), *i.e.* they are crystallographically equivalent. This result can readily be obtained if we consider the stabilizer of a disoriented domain state  $\mathbf{R}_1^+$ , which is  $I_{4/mmm}(\mathbf{R}_1^+) = 2_z/m_z$ . Then the number  $n_a^{\text{dis}}$  of disoriented ferroelastic domain states is given by

$$n_a^{\text{dis}} = [G : I_g(\mathbf{R}_1^+)] = |4_z/m_z m_x m_{xy}| : |2_z/m_z| = 16 : 4 = 4. \quad (3.4.3.69)$$

All these domain states appear in ferroelastic polydomain structures that contain coexisting lamellar structures (3.4.3.67) and (3.4.3.68).

Disoriented domain states in ferroelastic domain structures can be recognized by diffraction techniques (*e.g.* using an X-ray precession camera). The presence of these four disoriented domain states results in splitting of the diffraction spots of the high-symmetry tetragonal phase into four or two spots in the orthorhombic ferroelastic phase. This splitting is schematically depicted in Fig. 3.4.3.9. For more details see *e.g.* Shmyt'ko *et al.* (1987), Rosová *et al.* (1993), and Rosová (1999).