

3.4. DOMAIN STRUCTURES

Table 3.4.4.2. Action of four types of operations g on a twin $(S_1|n|S_2)$

Operation g keeps the orientation of the plane p unchanged.

| g | gS_1 | gS_2 | gn | $g(S_1 $ | $g S_2)$ | $g(S_1 n S_2) = (gS_1 gn gS_2)$ | Resulting twin |
|------------------------|--------|--------|------|----------|----------|-----------------------------------|----------------|
| f_{1j} | S_1 | S_2 | n | $(S_1 $ | $ S_2)$ | $(S_1 n S_2)$ | Initial twin |
| \underline{s}_{1j} | S_1 | S_2 | $-n$ | $ S_1)$ | $(S_2 $ | $(S_1 -n S_2) \equiv (S_2 n S_1)$ | Reversed twin |
| r_{1j}^* | S_2 | S_1 | n | $(S_2 $ | $ S_1)$ | $(S_2 n S_1)$ | Reversed twin |
| \underline{t}_{1j}^* | S_2 | S_1 | $-n$ | $ S_2)$ | $(S_1 $ | $(S_2 -n S_1) \equiv (S_1 n S_2)$ | Initial twin |

(4) An operation \underline{t}_{1j}^* which inverts n and simultaneously exchanges S_1 and S_2 . This operation, called the *non-trivial symmetry operation of a twin*, transforms the initial twin into $(S_2|-n|S_1)$, which is, according to (3.4.4.1), identical with the initial twin $(S_1|n|S_2)$. An operation of this type can be expressed as a product of a side-exchanging operation (underlined) and a state-exchanging operation (with a star), and will, therefore, be underlined and marked by a star. In Fig. 3.4.4.2, a non-trivial symmetry operation is for example the 180° rotation $\underline{2}_z^*$.

We note that the star and the underlining do not represent any operation; they are just suitable auxiliary labels that can be omitted without changing the result of the operation.

To find all trivial symmetry operations of the twin $(S_1|n|S_2)$, we recall that all symmetry operations that leave both S_1 and S_2 invariant constitute the symmetry group F_{1j} of the ordered domain pair (S_1, S_2) , $F_{1j} = F_1 \cap F_2$, where F_1 and F_2 are the symmetry groups of S_1 and S_2 , respectively. The sectional layer group of the plane p in group F_{1j} is (if we omit p)

$$\bar{F}_{1j} = \hat{F}_{1j} \cup \underline{s}_{1j}\hat{F}_{1j}. \tag{3.4.4.12}$$

The trivial (side-preserving) subgroup \hat{F}_{1j} assembles all trivial symmetry operations of the twin $(S_1|n|S_2)$. The left coset $\underline{s}_{1j}\hat{F}_{1j}$, where \underline{s}_{1j} is a side-reversing operation, contains all side-reversing operations of this twin. In our example $\hat{F}_{12} = \{1, m_z\}$ and $\underline{s}_{1j}\hat{F}_{1j} = \underline{m}_y\{1, m_z\} = \{\underline{m}_y, \underline{2}_z\}$ (see Fig. 3.4.4.2).

Similarly, the left coset $r_{1j}^*\hat{F}_{1j}$ contains all state-exchanging operations, and $\underline{t}_{1j}^*\hat{F}_{1j}$ all non-trivial symmetry operations of the twin $(S_1|n|S_2)$. In the illustrative example, $r_{1j}^*\hat{F}_{1j} = m_x^*\{1, m_z\} = \{m_x^*, \underline{2}_y^*\}$ and $\underline{t}_{1j}^*\hat{F}_{1j} = \underline{2}_z^*\{1, m_z\} = \{\underline{2}_z^*, \underline{1}^*\}$.

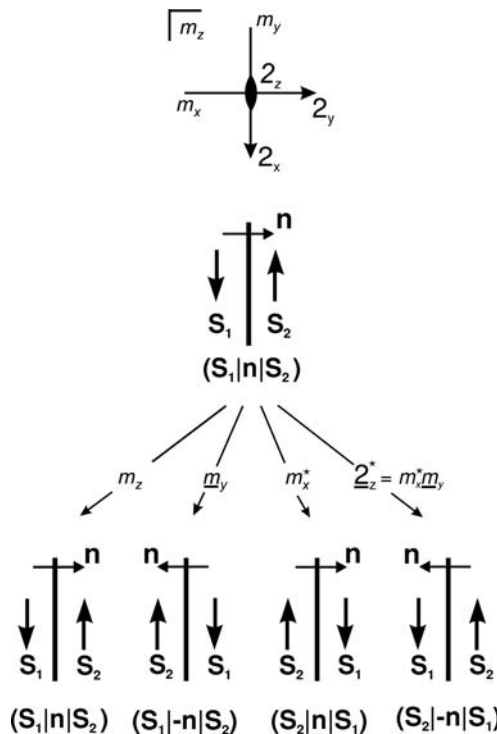


Fig. 3.4.4.2. A simple twin under the action of four types of operation that do not change the orientation of the wall plane p . Compare with Table 3.4.4.2.

The trivial group \hat{F}_{1j} and its three cosets constitute the sectional layer group \bar{J}_{1j} of the plane p in the symmetry group $J_{1j} = F_{1j} \cup g_{1j}^*F_{1j}$ of the unordered domain pair $\{S_1, S_2\}$,

$$\bar{J}_{1j} = \hat{J}_{1j} \cup \underline{s}_{1j}\hat{J}_{1j} = \hat{F}_{1j} \cup r_{1j}^*\hat{F}_{1j} \cup \underline{s}_{1j}\hat{F}_{1j} \cup \underline{t}_{1j}^*\hat{F}_{1j}, \tag{3.4.4.13}$$

where r_{1j}^* is an operation of the left coset $g_{1j}^*F_{1j}$ that leaves the normal n invariant and $\underline{t}_{1j}^* = \underline{s}_{1j}r_{1j}^*$.

Group \bar{J}_{1j} can be interpreted as a symmetry group of a *twin pair* $(S_1, S_2|n|S_2, S_1)$ consisting of a domain twin $(S_1|n|S_2)$ and a superposed reversed twin $(S_2|n|S_1)$ with a common wall plane p . This construct is analogous to a domain pair (dichromatic complex in bicrystallography) in which two homogeneous domain states S_1 and S_2 are superposed (see Section 3.4.3.1). In the same way as the group J_{1j} of domain pair $\{S_1, S_2\}$ is divided into two cosets with different results of the action on this domain pair, the symmetry group \bar{J}_{1j} of the twin pair can be decomposed into four cosets (3.4.4.13), each of which acts on a domain twin $(S_2|n|S_1)$ in a different way, as specified in Table 3.4.4.2.

We can associate with operations from each coset in (3.4.4.13) a label. If we denote operations from \hat{F}_{1j} without a label by e , underlining by a and star by b , then the multiplication of labels is expressed by the relations

$$a^2 = b^2 = e, \quad ab = ba. \tag{3.4.4.14}$$

The four different labels e, a, b, ab can be formally viewed as four colours, the permutation of which is defined by relations (3.4.3.14); then the group \bar{J}_{1j} can be treated as a four-colour layer group.

Since the symbol of a point group consists of generators from which any operation of the group can be derived by multiplication, one can derive from the international symbol of a sectional layer group, in which generators are supplied with adequate labels, the coset decomposition (3.4.4.13).

Thus for the domain pair $\{S_1, S_2\}$ in Fig. 3.4.4.2 with $J_{12} = m_x^*m_y m_z$ [see equation (3.4.3.18)] and $p(010)$ we get the sectional layer group $\bar{J}_{12}(010) = m_x^*\underline{m}_y m_z$. Operations of this group (besides generators) are $m_x^*\underline{m}_y = \underline{2}_z^*$, $\underline{m}_y m_z = \underline{2}_x$, $m_x^*m_z = \underline{2}_y^*$, $m_x^*\underline{2}_x = \underline{1}^*$.

All operations $g \in G$ that transform a twin into itself constitute the *symmetry group* $T_{1j}(n)$ (or in short T_{1j}) of the twin $(S_1|n|S_2)$. This is a layer group consisting of two parts:

$$T_{1j} = \hat{F}_{1j} \cup \underline{t}_{1j}^*\hat{F}_{1j}, \tag{3.4.4.15}$$

where \hat{F}_{1j} is a face group comprising all trivial symmetry operations of the twin and the left coset $\underline{t}_{1j}^*\hat{F}_{1j}$ contains all non-trivial operations of the twin that reverse the sides of the wall plane p and simultaneously exchange the states $(S_1$ and $S_2)$.

One can easily verify that the symmetry $T_{1j}(n)$ of the twin $(S_1|n|S_2)$ is equal to the symmetry $T_{j1}(n)$ of the reversed twin $(S_2|n|S_1)$,

$$T_{1j}(n) = T_{j1}(n). \tag{3.4.4.16}$$

Similarly, for sectional layer groups,

$$\bar{F}_{1j}(n) = \bar{F}_{j1}(n) \quad \text{and} \quad \bar{J}_{1j}(n) = \bar{J}_{j1}(n). \tag{3.4.4.17}$$