International Tables for Crystallography (2013). Vol. D, Table 3.4.4.2, p. 531.

Table 3.4.4.2. *Action of four types of operations g on a twin* $(\mathbf{S}_1|\mathbf{n}|\mathbf{S}_j)$

Operation g keeps the orientation of the plane p unchanged.

g	$g\mathbf{S}_1$	$g\mathbf{S}_j$	$g\mathbf{n}$	$g(\mathbf{S}_1\vert$	$g\vert\mathbf{S}_j)$	$g(\mathbf{S}_1\vert\mathbf{n}\vert\mathbf{S}_j) = (g\mathbf{S}_1\vert g\mathbf{n}\vert g\mathbf{S}_j)$	Resulting twin
f_{1j}	$\mathbf{S}_1$	$\mathbf{S}_j$	$\mathbf{n}$	$(\mathbf{S}_1\vert$	$\vert\mathbf{S}_j)$	$(\mathbf{S}_1\vert\mathbf{n}\vert\mathbf{S}_j)$	Initial twin
$\underline{s}_{1j}$	$\mathbf{S}_1$	$\mathbf{S}_j$	$-\mathbf{n}$	$\vert\mathbf{S}_1)$	$(\mathbf{S}_j\vert$	$(\mathbf{S}_1\vert-\mathbf{n}\vert\mathbf{S}_j) \equiv (\mathbf{S}_j\vert\mathbf{n}\vert\mathbf{S}_1)$	Reversed twin
$r^\star_{1j}$	$\mathbf{S}_j$	$\mathbf{S}_1$	$\mathbf{n}$	$(\mathbf{S}_j\vert$	$\vert\mathbf{S}_1)$	$(\mathbf{S}_j\vert\mathbf{n}\vert\mathbf{S}_1)$	Reversed twin
$\underline{t}^\star_{1j}$	$\mathbf{S}_j$	$\mathbf{S}_1$	$-\mathbf{n}$	$\vert\mathbf{S}_j)$	$(\mathbf{S}_1\vert$	$(\mathbf{S}_j\vert-\mathbf{n}\vert\mathbf{S}_1) \equiv (\mathbf{S}_1\vert\mathbf{n}\vert\mathbf{S}_j)$	Initial twin

(4) An operation $\underline{t}^\star_{1j}$ which inverts $\mathbf{n}$ and simultaneously exchanges $\mathbf{S}_1$ and $\mathbf{S}_j$. This operation, called the *non-trivial symmetry operation of a twin*, transforms the initial twin into $(\mathbf{S}_j|-\mathbf{n}|\mathbf{S}_1)$, which is, according to (3.4.4.1), identical with the initial twin $(\mathbf{S}_1|\mathbf{n}|\mathbf{S}_j)$. An operation of this type can be expressed as a product of a side-exchanging operation (underlined) and a state-exchanging operation (with a star), and will, therefore, be underlined and marked by a star. In Fig. 3.4.4.2, a non-trivial symmetry operation is for example the 180° rotation $\underline{2}^\star_z$.

We note that the star and the underlining do not represent any operation; they are just suitable auxiliary labels that can be omitted without changing the result of the operation.

To find all trivial symmetry operations of the twin $(\mathbf{S}_1|\mathbf{n}|\mathbf{S}_j)$, we recall that all symmetry operations that leave both $\mathbf{S}_1$ and $\mathbf{S}_j$ invariant constitute the symmetry group F_{1j} of the ordered domain pair $(\mathbf{S}_1, \mathbf{S}_j)$, $F_{1j} = F_1 \cap F_j$, where F_1 and F_j are the symmetry groups of $\mathbf{S}_1$ and $\mathbf{S}_j$, respectively. The sectional layer group of the plane p in group F_{1j} is (if we omit p)

$$\overline{\mathsf{F}}_{1j} = \widehat{\mathsf{F}}_{1j} \cup \underline{s}_{1j}\widehat{\mathsf{F}}_{1j}. \qquad (3.4.4.12)$$

The trivial (side-preserving) subgroup $\widehat{\mathsf{F}}_{1j}$ assembles all trivial symmetry operations of the twin $(\mathbf{S}_1|\mathbf{n}|\mathbf{S}_j)$. The left coset $\underline{s}_{1j}\widehat{\mathsf{F}}_{1j}$, where $\underline{s}_{1j}$ is a side-reversing operation, contains all side-reversing operations of this twin. In our example $\widehat{F}_{12} = \{1, m_z\}$ and $\underline{s}_{1j}\widehat{\mathsf{F}}_{1j} = \underline{m}_y\{1, m_z\} = \{\underline{m}_y, \underline{2}_x\}$ (see Fig. 3.4.4.2).

Similarly, the left coset $r^\star_{1j}\widehat{\mathsf{F}}_{1j}$ contains all state-exchanging operations, and $\underline{t}^\star_{1j}\widehat{\mathsf{F}}_{1j}$ all non-trivial symmetry operations of the twin $(\mathbf{S}_1|\mathbf{n}|\mathbf{S}_j)$. In the illustrative example, $r^\star_{1j}\widehat{\mathsf{F}}_{1j} = m^\star_x\{1, m_z\} = \{m^\star_x, 2^\star_y\}$ and $\underline{t}^\star_{1j}\widehat{\mathsf{F}}_{1j} = \underline{2}^\star_z\{1, m_z\} = \{\underline{2}^\star_z, \underline{\bar{1}}^\star\}$.

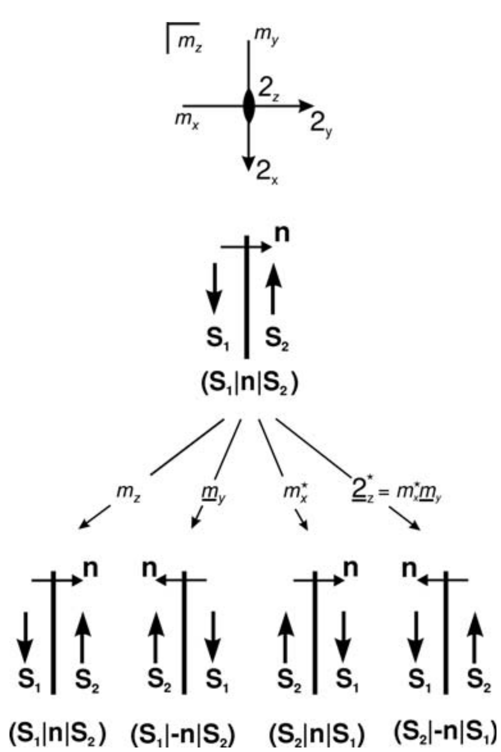

Fig. 3.4.4.2. A simple twin under the action of four types of operation that do not change the orientation of the wall plane p. Compare with Table 3.4.4.2.

The trivial group $\widehat{F}_{1j}$ and its three cosets constitute the sectional layer group $\overline{J}_{1j}$ of the plane p in the symmetry group $J_{1j} = F_{1j} \cup g^\star_{1j}F_{1j}$ of the unordered domain pair $\{\mathbf{S}_1, \mathbf{S}_j\}$,

$$\overline{\mathsf{J}}_{1j} = \widehat{\mathsf{J}}_{1j} \cup \underline{s}_{1j}\widehat{\mathsf{J}}_{1j} = \widehat{\mathsf{F}}_{1j} \cup r^\star_{1j}\widehat{\mathsf{F}}_{1j} \cup \underline{s}_{1j}\widehat{\mathsf{F}}_{1j} \cup \underline{t}^\star_{1j}\widehat{\mathsf{F}}_{1j}, \qquad (3.4.4.13)$$

where $r^\star_{1j}$ is an operation of the left coset $g^\star_{1j}F_{1j}$ that leaves the normal $\mathbf{n}$ invariant and $\underline{t}^*_{1j} = \underline{s}_{1j}r^\star_{1j}$.

Group $\overline{\mathsf{J}}_{1j}$ can be interpreted as a symmetry group of a *twin pair* $(\mathbf{S}_1, \mathbf{S}_j|\mathbf{n}|\mathbf{S}_j, \mathbf{S}_1)$ consisting of a domain twin $(\mathbf{S}_1|\mathbf{n}|\mathbf{S}_j)$ and a superposed reversed twin $(\mathbf{S}_j|\mathbf{n}|\mathbf{S}_1)$ with a common wall plane p. This construct is analogous to a domain pair (dichromatic complex in bicrystallography) in which two homogeneous domain states $\mathbf{S}_1$ and $\mathbf{S}_j$ are superposed (see Section 3.4.3.1). In the same way as the group J_{1j} of domain pair $\{\mathbf{S}_1, \mathbf{S}_j\}$ is divided into two cosets with different results of the action on this domain pair, the symmetry group $\overline{\mathsf{J}}_{1j}$ of the twin pair can be decomposed into four cosets (3.4.4.13), each of which acts on a domain twin $(\mathbf{S}_j|\mathbf{n}|\mathbf{S}_1)$ in a different way, as specified in Table 3.4.4.2.

We can associate with operations from each coset in (3.4.4.13) a label. If we denote operations from $\widehat{\mathsf{F}}_{1j}$ without a label by e, underlining by a and star by b, then the multiplication of labels is expressed by the relations

$$a^2 = b^2 = e, \quad ab = ba. \qquad (3.4.4.14)$$

The four different labels e, a, b, ab can be formally viewed as four colours, the permutation of which is defined by relations (3.4.3.14); then the group $\overline{\mathsf{J}}_{1j}$ can be treated as a four-colour layer group.

Since the symbol of a point group consists of generators from which any operation of the group can be derived by multiplication, one can derive from the international symbol of a sectional layer group, in which generators are supplied with adequate labels, the coset decomposition (3.4.4.13).

Thus for the domain pair $\{\mathbf{S}_1, \mathbf{S}_2\}$ in Fig. 3.4.4.2 with $J^\star_{12} = m^\star_x m_y m_z$ [see equation (3.4.3.18)] and $p(010)$ we get the sectional layer group $\overline{\mathsf{J}}_{12}(010) = m^\star_x\underline{m}_y m_z$. Operations of this group (besides generators) are $m^\star_x\underline{m}_y = \underline{2}^\star_z$, $\underline{m}_y m_z = \underline{2}_x$, $m^\star_x m_z = 2^\star_y$, $m^\star_x\underline{2}_x = \underline{\bar{1}}^\star$.

All operations $g \in G$ that transform a twin into itself constitute the *symmetry group* $\mathsf{T}_{1j}(\mathbf{n})$ (or in short T_{1j}) *of the twin* $(\mathbf{S}_1|\mathbf{n}|\mathbf{S}_j)$. This is a layer group consisting of two parts:

$$\mathsf{T}_{1j} = \widehat{\mathsf{F}}_{1j} \cup \underline{t}^\star_{1j}\widehat{\mathsf{F}}_{1j}, \qquad (3.4.4.15)$$

where $\widehat{\mathsf{F}}_{1j}$ is a face group comprising all trivial symmetry operations of the twin and the left coset $\underline{t}^*_{1j}\widehat{\mathsf{F}}_{1j}$ contains all non-trivial operations of the twin that reverse the sides of the wall plane p and simultaneously exchange the states ($\mathbf{S}_1$ and $\mathbf{S}_j$).

One can easily verify that the symmetry $\mathsf{T}_{1j}(\mathbf{n})$ of the twin $(\mathbf{S}_1|\mathbf{n}|\mathbf{S}_j)$ is equal to the symmetry $\mathsf{T}_{j1}(\mathbf{n})$ of the reversed twin $(\mathbf{S}_j|\mathbf{n}|\mathbf{S}_1)$,

$$\mathsf{T}_{1j}(\mathbf{n}) = \mathsf{T}_{j1}(\mathbf{n}). \qquad (3.4.4.16)$$

Similarly, for sectional layer groups,

$$\overline{\mathsf{F}}_{1j}(\mathbf{n}) = \overline{\mathsf{F}}_{j1}(\mathbf{n}) \text{ and } \overline{\mathsf{J}}_{1j}(\mathbf{n}) = \overline{\mathsf{J}}_{j1}(\mathbf{n}). \qquad (3.4.4.17)$$

references