$\overline{\mathsf{F}}_{1j} = \widehat{\mathsf{F}}_{1j} \cup \underline{s}_{1j}\widehat{\mathsf{F}}_{1j}$, the trivial layer group $\widehat{\mathsf{J}}_{1j} = \widehat{\mathsf{F}}_{1j} \cup r^{*}_{1j}\widehat{\mathsf{F}}_{1j}$ and the trivial layer group $\widehat{\mathsf{F}}_{1j}$.

An example of a symmetric reversible (SR) twin (and wall) is the twin $(\mathbf{S}_1[010]\mathbf{S}_2)$ in Fig. 3.4.4.2 with a non-trivial twinning operation $\underline{2}^\star_z$ and with reversing operations $\underline{m}_y$ and $m^\star_x$. The twin $(\mathbf{S}_1^+[\bar{1}10]\mathbf{S}_3^-)$ and reversed twin $(\mathbf{S}_3^-[\bar{1}10]\mathbf{S}_1^+)$ in Fig. 3.4.3.8 are symmetric and irreversible (SI) twins with a twinning operation $\underline{m}^\star_{xy}$; no reversing operations exist (walls are charged and charged walls are always irreversible, since a charge is invariant with respect to any transformation of the space). The twin $(\mathbf{S}_1^-[110]\mathbf{S}_3^+)$ and reversed twin $(\mathbf{S}_3^+[110]\mathbf{S}_1^-)$ in the same figure are asymmetric state-reversible twins with state-reversing operation $m^\star_{xy}$ and with no non-trivial twinning operation.

The same classification also applies to domain twins and walls in a microscopic description.

As in the preceding section, we shall now present separately the symmetries of non-ferroelastic simple domain twins ['twinning without a change of crystal shape (or form)'] and of ferroelastic simple domain twins ['twinning with a change of crystal shape (or form)'; Klassen-Neklyudova (1964), Indenbom (1982)].

### 3.4.4.4. *Non-ferroelastic domain twins and domain walls*

Compatibility conditions impose no restriction on the orientation of non-ferroelastic domain walls. Any of the non-ferroelastic domain pairs listed in Table 3.4.3.4 can be sectioned on any crystallographic plane $p$ and the sectional group $\overline{\mathsf{J}}_{1j}$ specifies the symmetry properties of the corresponding twin and domain wall. The analysis can be confined to one representative orientation of each class of equivalent planes, but a listing of all possible cases is too voluminous for the present article. We give, therefore, in Table 3.4.4.4 only possible symmetries $\mathsf{T}_{1j}$ and $\overline{\mathsf{J}}_{1j}$ of non-ferroelastic domain twins and walls, together with their classification, without specifying the orientation of the wall plane $p$.

Non-ferroelastic domain walls are usually curved with a slight preference for certain orientations (see Figs. 3.4.1.5 and 3.4.3.3). Such shapes indicate a weak anisotropy of the wall energy $\sigma$, *i.e.* small changes of $\sigma$ with the orientation of the wall. The situation is different in ferroelectric domain structures, where charged domain walls have higher energies than uncharged ones.

A small energetic anisotropy of non-ferroelastic domain walls is utilized in producing *tailored domain structures* (Newnham *et al.*, 1975). A required domain pattern in a non-ferroelastic ferroelectric crystal can be obtained by evaporating electrodes of a desired shape (*e.g.* stripes) onto a single-domain plate cut perpendicular to the spontaneous polarization $\mathbf{P}_0$. Subsequent poling by an electric field switches only regions below the electrodes and thus produces the desired antiparallel domain structure.

Periodically poled ferroelectric domain structures fabricated by this technique are used for example in quasi-phase-matching optical multipliers (see *e.g.* Shur *et al.*, 1999, 2001; Rosenman *et al.*, 1998). An example of such an *engineered domain structure* is presented in Fig. 3.4.4.3.

Anisotropic domain walls can also appear if the Landau free energy contains a so-called Lifshitz invariant (see Section 3.1.3.3), which lowers the energy of walls with certain orientations and can be responsible for the appearance of an incommensurate phase (see *e.g.* Dolino, 1985; Tolédano & Tolédano, 1987; Tolédano & Dmitriev, 1996; Strukov & Levanyuk, 1998). The irreversible character of domain walls in a commensurate phase of crystals also containing (at least theoretically) an incommensurate phase has been confirmed in the frame of phenomenological theory by Ishibashi (1992). The incommensurate structure in quartz that demonstrates such an anisotropy is discussed at the end of the next example.

Table 3.4.4.4. *Symmetries of non-ferroelastic domain twins and walls*

| $\mathsf{T}_{1j}$ | $\overline{\mathsf{J}}_{1j}$ | Classification |
|---|---|---|
| 1 | 1 | AI |
| 1 | $\underline{\bar{1}}$ | A$\underline{\mathrm{R}}$ |
| | $\underline{2}$ | A$\underline{\mathrm{R}}$ |
| | $2^\star$ | AR$^\star$ |
| | $m^\star$ | AR$^\star$ |
| $\bar{1}^\star$ | $\bar{1}^\star$ | SI |
| | $\underline{2}/m^\star$ | SR |
| | $2^\star/\underline{m}$ | SR |
| 2 | $2m^\star m^\star$ | AR$^\star$ |
| $\underline{2}^\star$ | $\underline{2}^\star$ | SI |
| | $\underline{2}^\star/m^\star$ | SR |
| | $2^\star\underline{2}^\star\underline{2}$ | SR |
| | $\underline{2}^\star\underline{mm}^\star$ | SR |
| $m$ | $m$ | AI |
| | $\underline{2}/m$ | A$\underline{\mathrm{R}}$ |
| | $2^\star mm^\star$ | AR$^\star$ |
| $\underline{m}^\star$ | $\underline{m}^\star$ | SI |
| | $2^\star/\underline{m}^\star$ | SR |
| | $\underline{m}^\star m^\star\underline{2}$ | SR |
| $\underline{2}^\star/m$ | $\underline{2}^\star/m$ | SI |
| | $m\underline{m}m^\star$ | SR |
| $2/\underline{m}^\star$ | $2/\underline{m}^\star$ | SI |
| | $m^\star m^\star\underline{m}^\star$ | SR |
| | $4^\star/\underline{m}^\star$ | SR |
| $2\underline{2}^\star\underline{2}^\star$ | $2\underline{2}^\star\underline{2}^\star$ | SI |
| | $\underline{m}m^\star m^\star$ | SR |
| | $4^\star\underline{22}^\star$ | SR |
| | $\underline{\bar{4}2}^\star m^\star$ | SR |
| $\underline{m}^\star m\underline{2}^\star$ | $\underline{m}^\star m\underline{2}^\star$ | SI |
| | $\underline{m}^\star mm^\star$ | SR |
| $mm\underline{m}^\star$ | $mm\underline{m}^\star$ | SI |
| | $4^\star/\underline{m}^\star m^\star m$ | SR |
| 4 | $4m^\star m^\star$ | AR$^\star$ |
| $\underline{\bar{4}}^\star$ | $\underline{\bar{4}}^\star\underline{2}m^\star$ | SR |
| $4/\underline{m}^\star$ | $4/\underline{m}^\star$ | SI |
| | $4/\underline{m}^\star m^\star m^\star$ | SR |
| $4\underline{2}^\star\underline{2}^\star$ | $4\underline{2}^\star\underline{2}^\star$ | SI |
| | $4/\underline{m}m^\star m^\star$ | SR |
| $\underline{\bar{4}}^\star\underline{2}^\star m$ | $4^\star/\underline{m}m^\star m$ | SR |
| $4/\underline{m}^\star mm$ | $4/\underline{m}^\star mm$ | SI |
| 3 | $3m^\star$ | AR$^\star$ |
| | $6^\star$ | AR$^\star$ |
| $\underline{\bar{3}}^\star$ | $\underline{\bar{3}}^\star$ | SI |
| | $\underline{\bar{3}}^\star m^\star$ | SR |
| | $6^\star/\underline{m}$ | SR |
| $3m$ | $6^\star mm^\star$ | AR$^\star$ |
| $3\underline{2}^\star$ | $3\underline{2}^\star$ | SI |
| | $\underline{\bar{3}}m^\star$ | SR |
| | $6^\star\underline{22}^\star$ | SR |
| | $\underline{\bar{6}2}^\star m^\star$ | SR |
| $\underline{\bar{3}}^\star m$ | $\underline{\bar{3}}^\star m$ | SI |
| | $6^\star/\underline{m}mm^\star$ | SR |
| 6 | $6m^\star m^\star$ | AR$^\star$ |
| $\underline{\bar{6}}^\star$ | $\underline{\bar{6}}^\star$ | SI |
| | $6^\star/\underline{m}^\star$ | SR |
| | $\underline{\bar{6}}^\star\underline{2}m^\star$ | SR |
| $6/\underline{m}^\star$ | $6/\underline{m}^\star$ | SI |
| | $6/\underline{m}^\star m^\star m^\star$ | SR |
| $6\underline{2}^\star\underline{2}^\star$ | $6\underline{2}^\star\underline{2}^\star$ | SI |
| | $6/\underline{m}m^\star m^\star$ | SR |
| $\underline{\bar{6}}^\star m\underline{2}^\star$ | $\underline{\bar{6}}^\star m\underline{2}^\star$ | SI |
| | $6^\star/\underline{m}^\star mm^\star$ | SR |
| $6/\underline{m}^\star mm$ | $6/\underline{m}^\star mm$ | SI |

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