

1.2. GUIDE TO THE USE OF THE SUBPERIODIC GROUP TABLES

Column 10: symbols listed by Lockwood & Macmillan (1978).

Column 11: symbols listed by Shubnikov & Koptsik (1974).

Sets of symbols which are of a non-Hermann–Mauguin (international) type are the set of symbols of the ‘black and white’ symmetry type (column 3) and the sets of symbols in columns 6 and 7. The sets of symbols in columns 4, 5 and 11 do not follow the sequence of symmetry directions used for two-dimensional space groups. The sets of symbols in columns 3, 4, 5 and 10 do not use a lower-case script \not{r} to denote a one-dimensional lattice. The set of symbols in column 9 uses parentheses and square brackets to denote specific symmetry directions. The symbol g is used in Part 1 to denote a glide line, a standard symbol for two-dimensional space groups (*IT A*, 1983). A letter identical with a basis-vector symbol, e.g. a or c , is not used to denote a glide line, as is done in the symbols of columns 5, 6, 7, 9 and 11, as such a letter is a standard notation for a three-dimensional glide plane (*IT A*, 1983).

Columns 2 and 3 show the isomorphism between frieze groups and one-dimensional magnetic space groups. The one-dimensional space groups are denoted by $\not{r}1$ and $\not{r}\bar{1}$. The list of symbols in column 3, on replacing r with \not{r} , is the list of one-dimensional magnetic space groups. The isomorphism between these two sets of groups interchanges the elements $\bar{1}$ and $1'$ of the one-dimensional magnetic space groups and, respectively, the elements m_x and m_y , mirror lines perpendicular to the $[10]$ and $[01]$ directions, of the frieze groups.

1.2.17.2. Rod groups

A list of sets of symbols for the rod groups is given in Table 1.2.17.2. The information provided in the columns of this table is as follows:

Columns 1 and 2: sequential numbering and symbols used in Part 3.

Columns 3 and 4: sequential numbering and symbols listed by Bohm & Dornberger-Schiff (1966, 1967).

Columns 5, 6 and 7: sequential numbering and two sets of symbols listed by Shubnikov & Koptsik (1974).

Column 8: symbols listed by Opechowski (1986).

Column 9: symbols listed by Niggli (Chapuis, 1966).

Sets of symbols which are of a non-Hermann–Mauguin (international) type are the set of symbols in column 6 and the Niggli-type set of symbols in column 9. The set of symbols in column 8 does not use the lower-case script letter \not{r} , as does *IT A* (1983), to denote a one-dimensional lattice. The order of the characters indicating symmetry elements in the set of symbols in column 7 does not follow the sequence of symmetry directions used for three-dimensional space groups. The set of symbols in column 4 have the characters indicating symmetry elements along non-lattice directions enclosed in parentheses, and do not use a lower-case script letter to denote the one-dimensional lattice. Lastly, the set of symbols in column 4, without the parentheses and with the one-dimensional lattice denoted by a lower-case script \not{r} , are identical with the symbols in Part 3, or in some cases are the second setting of rod groups whose symbols are given in Part 3. These second-setting symbols are included in the symmetry diagrams of the rod groups.

1.2.17.3. Layer groups

A list of sets of symbols for the layer groups is given in Table 1.2.17.3. The information provided in the columns of this table is as follows:

Columns 1 and 2: sequential numbering and symbols used in Part 4.

Columns 3 and 4: sequential numbering and symbols listed by Wood (1964*a,b*) and Litvin & Wike (1991).

Columns 5 and 6: sequential numbering and symbols listed by Bohm & Dornberger-Schiff (1966, 1967).

Columns 7 and 8: sequential numbering and symbols listed by Shubnikov & Koptsik (1974) and Vainshtein (1981).

Column 9: symbols listed by Holser (1958).

Column 10: sequential numbering listed by Weber (1929).

Column 11: symbols listed by Hermann (1929*a,b*).

Column 12: symbols listed by Alexander & Herrmann (1929*a,b*).

Column 13: symbols listed by Niggli (Wood, 1964*a,b*).

Column 14: symbols listed by Shubnikov & Koptsik (1974).

Columns 15 and 16: symbols listed by Aroyo & Wondratschek (1987).

Column 17: symbols listed by Belov *et al.* (1957).

Columns 18 and 19: symbols and sequential numbering listed by Belov & Tarkhova (1956*a,b*).

Columns 20 and 21: symbols listed by Cochran as listed, respectively, by Cochran (1952) and Belov & Tarkhova (1956*a,b*).

Column 22: symbols listed by Opechowski (1986).

Column 23: symbols listed by Grunbaum & Shephard (1987).

Column 24: symbols listed by Woods (1935*a,b,c*, 1936).

Column 25: symbols listed by Coxeter (1986).

There is also a notation for layer groups, introduced by Janovec (1981), in which all elements in the group symbol which change the direction of the normal to the plane containing the translations are underlined, e.g. $p4/\underline{m}$. However, we know of no listing of all layer-group types in this notation.

Sets of symbols which are of a non-Hermann–Mauguin (international) type are the sets of symbols of the Schoenflies type (columns 11 and 12) and symbols of the ‘black and white’ symmetry type (columns 16, 17, 18, 20, 21, 22, 24 and 25). Additional non-Hermann–Mauguin (international) type sets of symbols are those in columns 14 and 23.

Sets of symbols which do not begin with a letter indicating the lattice centring type are the sets of symbols of the Niggli type (columns 13 and 15). The order of the characters indicating symmetry elements in the sets of symbols in columns 4 and 9 does not follow the sequence of symmetry directions used for three-dimensional space groups. The set of symbols in column 6 uses parentheses to denote a symmetry direction which is not a lattice direction. In addition, the set of symbols in column 6 uses upper-case letters to denote the two-dimensional lattice of the layer group, where as in *IT A* (1983) upper-case letters denote three-dimensional lattices.

The symbols in column 8 are either identical with or, in some monoclinic and orthorhombic cases, are the second-setting or alternative-cell-choice symbols of the layer groups whose symbols are given in Part 4. These second-setting and alternative-cell-choice symbols are included in the symmetry diagrams of the layer groups.

The isomorphism between layer groups and two-dimensional magnetic space groups can be seen in Table 1.2.17.3. The set of symbols which we use for layer groups is given in column 2. The sets of symbols in columns 16, 17 and 22 are sets of symbols for the two-dimensional magnetic space groups. The basic relationship between these two sets of groups is the interchanging of the magnetic symmetry element $1'$ and the layer symmetry element m_z . A detailed discussion of the relationship between these two sets of groups has been given by Opechowski (1986).

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