

## 1. SUBPERIODIC GROUP TABLES: FRIEZE-GROUP, ROD-GROUP AND LAYER-GROUP TYPES

Table 1.2.17.2. Rod-group symbols (cont.)

	1	2	3	4	5	6	7	8	9
	67	$\mu 6_5 22$	69	$P6_5(22)$	75	$(a) \cdot 6_5 : 2$	$p6_5 22$	$r6_5 22$	$6_5 P22$
	68	$\mu 6 mm$	64	$P6(mm)$	43	$(a) \cdot 6 \cdot m$	$p6 mm$	$r6 mm$	$6 mm P1$
	69	$\mu 6 cc$	65	$P6(cc)$	44	$(a) \cdot 6 \cdot \bar{a}$	$p6 aa$	$r6 cc$	$6 gg P1$
	70	$\mu 6_3 mc$	66	$P6_3(cm)$	45	$(a) \cdot 6_3 \cdot m$	$p6_3 ma$	$r6_3 mc$	$6_3 mg P1$
	71	$\mu 6 m 2$	54	$P6(m2)$	51	$(a) \cdot m \cdot 3 : m$	$p6 m 2$	$\bar{r}6 m 2$	$3 m P m 2$
	72	$\mu 6 c 2$	55	$P6(c2)$	52	$(a) \cdot \bar{a} \cdot 3 : m$	$p\bar{6} a 2$	$\bar{r}6 c 2$	$3 g P m 2$
	73	$\mu 6 / m m m$	73	$P6/m(2/m2/m)$	56	$(a) \cdot m \cdot 6 : m$	$p6 / m m m$	$r6 / m 2 / m 2 / m$	$6 m m P m$
	74	$\mu 6 / m c c$	74	$P6/m(2/c2/c)$	57	$(a) \cdot \bar{a} \cdot 6 : m$	$p6 / m a a$	$r6 / m 2 / c 2 / c$	$6 g g P m$
	75	$\mu 6_3 / m m c$	75	$P6_3/m(2/c2/m)$	58	$(a) \cdot m \cdot 6_3 : m$	$p6_3 / m m a$	$r6_3 / m 2 / m 2 / c$	$6_3 m g P m$

Example: **G**: Layer group  $p2_1/m11$  (L15)

Minimal non-isotypic non-enantiomorphic supergroups:

**I** [2]  $pmam$ ; [2]  $pmma$ ; [2]  $pbma$ ; [2]  $pmmn$

**II** [2]  $c2/m11$ ; [2]  $p2/m11$  ( $2a' = a$ )

Block **I** lists [2]  $pmam$ , [2]  $pmma$  and [2]  $pmmn$ . Looking up the subgroup data of these three groups one finds [2]  $p2_1/m11$ . Block **I** also lists [2]  $pbma$ . Looking up the subgroup data of this group one finds [2]  $p12_1/m1$  ( $p2_1/m11$ ). This shows that the setting of  $pbma$  does not correspond to that of  $p2_1/m11$  but rather to  $p12_1/m1$ . To obtain the supergroup **H** referred to the basis of  $p2_1/m11$ , the basis vectors **a** and **b** must be interchanged. This changes  $pbma$  to  $pmba$ , which is the correct symbol of the supergroup of  $p2_1/m11$ .

Block **II** contains two entries: the first where the conventional cells are the same with the supergroup having additional centring translations, and the second where the conventional cell of the supergroup is smaller than that of the original subperiodic group.

#### 1.2.15.4. Minimal isotypic supergroups and enantiomorphic supergroups of lowest index

No data are listed for supergroups **IIc**, because they can be derived directly from the corresponding data of subgroups **IIc**.

Example: **G**: Rod group  $\mu 4_2/m$  (R29)

The maximal isotypic subgroup of lowest index of  $\mu 4_2/m$  is found in block **IIc**: [3]  $\mu 4_2/m$  ( $c' = 3c$ ). By interchanging  $c'$  and **c**, one obtains the minimal isotypic supergroup of lowest index, i.e. [3]  $\mu 4_2/m$  ( $3c' = c$ ).

#### 1.2.16. Nomenclature

There exists a wide variety of nomenclature for layer, rod and frieze groups (Holser, 1961). Layer-group nomenclature includes *zweidimensionale Raumgruppen* (Alexander & Herrmann, 1929a,b), *Ebenengruppen* (Weber, 1929), *Netzgruppen* (Hermann, 1929a), *net groups* (IT, 1952; Opechowski, 1986), *reversal space groups in two dimensions* (Cochran, 1952), *plane groups in three dimensions* (Dornberger-Schiff, 1956, 1959; Belov, 1959), *black and white space groups in two dimensions* (Mackay, 1957), *(two-sided) plane groups* (Holser, 1958), *Schichtgruppen* (Niggli, 1959; Chapuis, 1966), *diperiodic groups in three dimensions* (Wood, 1964a,b), *layer space groups* (Shubnikov & Koptsik, 1974), *layer groups* (Köhler, 1977; Koch & Fischer, 1978; Vainshtein, 1981; Goodman, 1984; Litvin, 1989), *two-dimensional (subperiodic) groups in three-dimensional space* (Brown et al., 1978) and *plane space groups in three dimensions* (Grell et al., 1989).

Rod-group nomenclature includes *Kettengruppen* (Hermann, 1929a,b), *eindimensionalen Raumgruppen* (Alexander, 1929, 1934), *(crystallographic) line groups in three dimensions* (IT, 1952; Opechowski, 1986), *rod groups* (Belov, 1956; Vujicic et al., 1977; Köhler, 1977; Koch & Fischer, 1978), *Balkengruppen*

(Niggli, 1959; Chapuis, 1966), *stem groups* (Galyarskii & Zamorzaev, 1965), *linear space groups* (Bohm & Dornberger-Schiff, 1966) and *one-dimensional (subperiodic) groups in three dimensions* (Brown et al., 1978).

Frieze-group nomenclature includes *Bortenornamente* (Speiser, 1927), *Bandgruppen* (Niggli, 1959), *line groups (borders) in two dimensions* (IT, 1952), *line groups in a plane* (Belov, 1956), *eindimensionale 'zweifarbige' Gruppen* (Nowacki, 1960), *groups of one-sided bands* (Shubnikov & Koptsik, 1974), *ribbon groups* (Köhler, 1977), *one-dimensional (subperiodic) groups in two-dimensional space* (Brown et al., 1978) and *groups of borders* (Vainshtein, 1981).

#### 1.2.17. Symbols

The following general criterion was used in selecting the sets of symbols for the subperiodic groups: *consistency with the symbols used for the space groups given in IT A* (1983). Specific criteria following from this general criterion are as follows:

(1) The symbols of subperiodic groups are to be of the Hermann-Mauguin (international) type. This is the type of symbol used for space groups in IT A (1983).

(2) A symbol of a subperiodic group is to consist of a letter indicating the lattice centring type followed by a set of characters indicating symmetry elements. This is the format of the Hermann-Mauguin (international) space-group symbols in IT A (1983).

(3) The sets of symmetry directions and their sequences in the symbols of the subperiodic groups are those of the corresponding space groups. Layer and rod groups are three-dimensional subperiodic groups of the three-dimensional space groups, and frieze groups are two-dimensional subperiodic groups of the two-dimensional space groups. Consequently, the symmetry directions and sequence of the characters indicating symmetry elements in layer and rod groups are those of the three-dimensional space groups; in frieze groups, they are those of the two-dimensional space groups, see Table 1.2.4.1 above and Table 2.4.1 of IT A (1983). Layer groups appear as subgroups of three-dimensional space groups, as factor groups of three-dimensional reducible space groups (Kopský, 1986, 1988, 1989a,b, 1993; Fuksa & Kopský, 1993) and as the symmetries of planes which transect a crystal of a given three-dimensional space-group symmetry. For example, the layer group  $pmm2$  is a subgroup of the three-dimensional space group  $Pmm2$ ; is isomorphic to the factor group  $Pmm2/T_z$  of the three-dimensional space group  $Pmm2$ , where  $T_z$  is the translational subgroup of all translations along the  $z$  axis; and is the symmetry of the plane transecting a crystal of three-dimensional space-group symmetry  $Pmm2$ , perpendicular to the  $z$  axis, at  $z = 0$ . In these examples, the symbols for the three-dimensional space group and the related subperiodic layer group differ only in the letter indicating the lattice type.

A survey of sets of symbols that have been used for the subperiodic groups is given below. Considering these sets of