Table 1.2.17.1. *Frieze-group symbols*

	1	2	3	4	5	6	7	8	9	10	11
Oblique	1	$\not r1$	$r1$	$r1$	$r111$	(a)	t	1	$p[1](1)1$	$r1$	$\not r1$
	2	$\not r211$	$r\bar{1}'$	$r112$	$r112$	$(a) : 2$	$t : 2$	5	$p[2](1)1$	$r2$	$\not r112$
Rectangular	3	$\not r1m1$	$r\bar{1}$	$r1m$	$rm11$	$(a) : m$	$t : m$	3	$p[1](1)m$	$r1m$	$\not rm11$
	4	$\not r11m$	$r11'$	rm	$r1m1$	$(a) \cdot m$	$t \cdot m$	2	$p[1](m)1$	$r11m$	$\not r1m1$
	5	$\not r11g$	$r_2 1$	rg	$r1c1$	$(a) \cdot \bar{a}$	$t \cdot a$	4	$p[1](c)1$	$r11g$	$\not r1a1$
	6	$\not r2mm$	$r\bar{1}1'$	$rmm2$	$rmm2$	$(a) : 2 \cdot m$	$t : 2 \cdot m$	6	$p[2](m)m$	$r2mm$	$\not rmm2$
	7	$\not r2mg$	$r_2\bar{1}$	$rgm2$	$rmc2$	$(a) : 2 \cdot \bar{a}$	$t : 2 \cdot a$	7	$p[2](c)m$	$r2mg$	$\not rma2$

(Niggli, 1959; Chapuis, 1966), *stem groups* (Galyarskii & Zamorzaev, 1965*a*,*b*), *linear space groups* (Bohm & Dornberger-Schiff, 1966) and *one-dimensional* (*subperiodic*) *groups in three dimensions* (Brown *et al.*, 1978).

Frieze-group nomenclature includes *Bortenornamente* (Speiser, 1927), *Bandgruppen* (Niggli, 1959), *line groups* (*borders*) *in two dimensions* (*IT*, 1952), *line groups in a plane* (Belov, 1956), *eindimensionale 'zweifarbige' Gruppen* (Nowacki, 1960), groups of *one-sided bands* (Shubnikov & Koptsik, 1974), *ribbon groups* (Köhler, 1977), *one-dimensional* (*subperiodic*) *groups in two-dimensional space* (Brown *et al.*, 1978) and *groups of borders* (Vainshtein, 1981).

1.2.17. Symbols

The following general criterion was used in selecting the sets of symbols for the subperiodic groups: *consistency with the symbols used for the space groups given in IT* A (2005). Specific criteria following from this general criterion are as follows:

(1) The symbols of subperiodic groups are to be of the Hermann–Mauguin (international) type. This is the type of symbol used for space groups in *IT* A (2005).

(2) A symbol of a subperiodic group is to consist of a letter indicating the lattice centring type followed by a set of characters indicating symmetry elements. This is the format of the Hermann–Mauguin (international) space-group symbols in *IT* A (2005).

(3) The sets of symmetry directions and their sequences in the symbols of the subperiodic groups are those of the corresponding space groups. Layer and rod groups are three-dimensional subperiodic groups of the three-dimensional space groups, and frieze groups are two-dimensional subperiodic groups of the two-dimensional space groups. Consequently, the symmetry directions and sequence of the characters indicating symmetry elements in layer and rod groups are those of the three-dimensional space groups; in frieze groups, they are those of the two-dimensional space groups, see Table 1.2.4.1 above and Table 2.2.4.1 of *IT* A (2005). Layer groups appear as subgroups of three-dimensional space groups, as factor groups of three-dimensional reducible space groups (Kopský, 1986, 1988, 1989*a*,*b*, 1993; Fuksa & Kopský, 1993) and as the symmetries of planes which transect a crystal of a given three-dimensional space-group symmetry. For example, the layer group *pmm*2 is a subgroup of the three-dimensional space group *Pmm*2; is isomorphic to the factor group $Pmm2/T_z$ of the three-dimensional space group *Pmm*2, where T_z is the translational subgroup of all translations along the z axis; and is the symmetry of the plane transecting a crystal of three-dimensional space-group symmetry *Pmm*2, perpendicular to the z axis, at $z = 0$. In these examples, the symbols for the three-dimensional space group and the related subperiodic layer group differ only in the letter indicating the lattice type.

A survey of sets of symbols that have been used for the subperiodic groups is given below. Considering these sets of symbols in relation to the above criteria leads to the sets of symbols for subperiodic groups used in Parts 2, 3 and 4.

1.2.17.1. Frieze groups

A list of sets of symbols for the frieze groups is given in Table 1.2.17.1. The information provided in this table is as follows:

Columns 1 and 2: sequential numbering and symbols used in Part 2.

Columns 3, 4 and 5: symbols listed by Opechowski (1986).

Column 6: symbols listed by Shubnikov & Koptsik (1974).

Column 7: symbols listed by Vainshtein (1981).

Columns 8 and 9: sequential numbering and symbols listed by Bohm & Dornberger-Schiff (1967).

Column 10: symbols listed by Lockwood & Macmillan (1978).

Column 11: symbols listed by Shubnikov & Koptsik (1974).

Sets of symbols which are of a non-Hermann–Mauguin (international) type are the set of symbols of the 'black and white' symmetry type (column 3) and the sets of symbols in columns 6 and 7. The sets of symbols in columns 4, 5 and 11 do not follow the sequence of symmetry directions used for two-dimensional space groups. The sets of symbols in columns 3, 4, 5 and 10 do not use a lower-case script $\not r$ to denote a one-dimensional lattice. The set of symbols in column 9 uses parentheses and square brackets to denote specific symmetry directions. The symbol *g* is used in Part 1 to denote a glide line, a standard symbol for two-dimensional space groups (*IT* A, 2005). A letter identical with a basis-vector symbol, *e.g.* *a* or *c*, is not used to denote a glide line, as is done in the symbols of columns 5, 6, 7, 9 and 11, as such a letter is a standard notation for a three-dimensional glide plane (*IT* A, 2005).

Columns 2 and 3 show the isomorphism between frieze groups and one-dimensional magnetic space groups. The one-dimensional space groups are denoted by $\not r1$ and $\not r\bar{1}$. The list of symbols in column 3, on replacing r with $\not r$, is the list of one-dimensional magnetic space groups. The isomorphism between these two sets of groups interexchanges the elements $\bar{1}$ and $1'$ of the one-dimensional magnetic space groups and, respectively, the elements m_x and m_y, mirror lines perpendicular to the [10] and [01] directions, of the frieze groups.

1.2.17.2. Rod groups

A list of sets of symbols for the rod groups is given in Table 1.2.17.2. The information provided in the columns of this table is as follows:

Columns 1 and 2: sequential numbering and symbols used in Part 3.

Columns 3 and 4: sequential numbering and symbols listed by Bohm & Dornberger-Schiff (1966, 1967).

Columns 5, 6 and 7: sequential numbering and two sets of symbols listed by Shubnikov & Koptsik (1974).

Table 1.2.17.2. *Rod-group symbols*

	1	2	3	4	5	6	7	8	9
Triclinic	1	$\not{1}$	1	$P(11)1$	1	$(a)\cdot 1$	$p1$	$r1$	$1P1$
	2	$\not{1}\bar{1}$	2	$P(\bar{1}\bar{1})\bar{1}$	7	$(a)\cdot\bar{1}$	$p\bar{1}$	$r\bar{1}$	$1P\bar{1}$
Monoclinic/inclined	3	$\not{2}11$	6	$P(12)1$	2	$(a):2$	$p112$	$r112$	$1P2$
	4	$\not{m}11$	3	$P(1m)1$	22	$(a)\cdot m$	$p11m$	$r1m1$	$mP1$
	5	$\not{c}11$	5	$P(1c)1$	24	$(a)\cdot\bar{a}$	$p11a$	$r1c1$	$gP1$
	6	$\not{2}/m11$	9	$P(12/m)1$	25	$(a):2:m$	$p112/m$	$r12/m1$	$mP2$
	7	$\not{2}/c11$	12	$P(12/c)1$	28	$(a):2:\bar{a}$	$p112/a$	$r12/c1$	$gP2$
Monoclinic/orthogonal	8	$\not{1}12$	7	$P(11)2$	3	$(a)\cdot 2$	$p211$	$r211$	$2P1$
	9	$\not{1}12_1$	8	$P(11)2_1$	8	$(a)\cdot 2_1$	$p2_1$	$r2_1$	2_1P1
	10	$\not{1}1m$	4	$P(11)m$	23	$(a):m$	$pm11$	$rm11$	$1Pm$
	11	$\not{1}12/m$	10	$P(11)2/m$	26	$(a)\cdot 2:m$	$p2/m11$	$r2/m11$	$2Pm$
	12	$\not{1}12_1/m$	11	$P(11)2_1/m$	27	$(a)\cdot 2_1:m$	$p2_1/m11$	$r2_1/m11$	2_1Pm
Orthorhombic	13	$\not{2}22$	18	$P(22)2$	61	$(a)\cdot 2:2$	$p222$	$r222$	$2P22$
	14	$\not{2}22_1$	19	$P(22)2_1$	62	$(a)\cdot 2_1:2$	$p2_122$	$r2_122$	2_1P22
	15	$\not{m}m2$	13	$P(mm)2$	34	$(a)\cdot 2\cdot m$	$p2mm$	$r2mm$	$2mmP1$
	16	$\not{c}c2$	16	$P(cc)2$	35	$(a)\cdot 2\cdot\bar{a}$	$p2aa$	$r2cc$	$2ggP1$
	17	$\not{m}c2_1$	15	$P(mc)2_1$	36	$(a)\cdot 2_1\cdot m$	$p2_1ma$	$r2_1mc$	2_1mgP1
	18	$\not{2}mm$	14	$P(2m)m$	33	$(a):2\cdot m$	$pmma$	$rmm2$	$mPm2$
	19	$\not{2}cm$	17	$P(2c)m$	37	$(a):2\cdot\bar{a}$	$pma2$	$rmc2$	$gPm2$
	20	$\not{m}mm$	20	$P(2/m2/m)2/m$	46	$(a)\cdot m\cdot 2:m$	$pmmm$	$r2/m2/m2/m$	$mmPm$
	21	$\not{c}cm$	21	$P(2/c2/c)2/m$	47	$(a)\cdot\bar{a}\cdot 2:m$	$pmaa$	$r2/m2/c2/c$	$ggPm$
	22	$\not{m}cm$	22	$P(2/m2/c)2_1/m$	48	$(a)\cdot m\cdot 2_1:m$	$pmma$	$r2_1/m2/m2/c$	$mgPm$
Tetragonal	23	$\not{4}$	26	$P4(11)$	5	$(a)\cdot 4$	$p4$	$r4$	$4P1$
	24	$\not{4}_1$	27	$P4_1(11)$	11	$(a)\cdot 4_1$	$p4_1$	$r4_1$	4_1P1
	25	$\not{4}_2$	28	$P4_2(11)$	12	$(a)\cdot 4_2$	$p4_2$	$r4_2$	4_2P1
	26	$\not{4}_3$	29	$P4_3(11)$	13	$(a)\cdot 4_3$	$p4_3$	$r4_3$	4_3P1
	27	$\not{4}\bar{4}$	23	$P\bar{4}(11)$	20	$(a)\cdot\bar{4}$	$p\bar{4}$	$r\bar{4}$	$1P\bar{4}$
	28	$\not{4}/m$	30	$P4/m(11)$	29	$(a)\cdot 4:m$	$p4/m$	$r4/m$	$4Pm$
	29	$\not{4}_2/m$	31	$P4_2/m(11)$	30	$(a)\cdot 4_2:m$	$p4_2/m$	$r4_2/m$	4_2Pm
	30	$\not{4}22$	35	$P4(22)$	66	$(a)\cdot 4:2$	$p422$	$r422$	$4P22$
	31	$\not{4}_122$	36	$P4_1(22)$	67	$(a)\cdot 4_1:2$	$p4_122$	$r4_122$	4_1P22
	32	$\not{4}_222$	37	$P4_2(22)$	68	$(a)\cdot 4_2:2$	$p4_222$	$r4_222$	4_2P22
	33	$\not{4}_322$	38	$P4_3(22)$	69	$(a)\cdot 4_3:2$	$p4_322$	$r4_322$	4_3P22
	34	$\not{4}mm$	32	$P4(mm)$	40	$(a)\cdot 4\cdot m$	$p4mm$	$r4mm$	$4mmP1$
	35	$\not{4}_2cm$	33	$P4_2(cm)$	42	$(a)\cdot 4_2\cdot m$	$p4_2ma$	$r4_2mc$	4_2mgP1
	36	$\not{4}cc$	34	$P4(cc)$	41	$(a)\cdot 4\cdot\bar{a}$	$p4aa$	$r4cc$	$4ggP1$
	37	$\not{4}\bar{4}2m$	24	$P\bar{4}(2m)$	49	$(a)\cdot\bar{4}\cdot m$	$p\bar{4}2m$	$r\bar{4}m2$	$mP\bar{4}2$
	38	$\not{4}\bar{4}2c$	25	$P\bar{4}(2c)$	50	$(a)\cdot\bar{4}\cdot\bar{a}$	$p\bar{4}2a$	$r\bar{4}c2$	$gP\bar{4}2$
	39	$\not{4}/mmm$	39	$P4/m(2/m2/m)$	53	$(a)\cdot m\cdot 4:m$	$p4/mmm$	$r4/m2/m2/m$	$4mmPm$
	40	$\not{4}/mmc$	40	$P4/m(2/c2/c)$	54	$(a)\cdot\bar{a}\cdot 4:m$	$p4/maa$	$r4/m2/c2/c$	$4ggPm$
	41	$\not{4}_2/mmc$	41	$P4_2/m(2/m2/c)$	55	$(a)\cdot m\cdot 4_2:m$	$p4_2/mma$	$r4_2/m2/m2/c$	4_2mgPm
Trigonal	42	$\not{3}$	42	$P3(11)$	4	$(a)\cdot 3$	$p3$	$r3$	$3P1$
	43	$\not{3}_1$	43	$P3_1(11)$	9	$(a)\cdot 3_1$	$p3_1$	$r3_1$	3_1P1
	44	$\not{3}_2$	44	$P3_2(11)$	10	$(a)\cdot 3_2$	$p3_2$	$r3_2$	3_2P1
	45	$\not{3}\bar{3}$	45	$P\bar{3}(11)$	19	$(a)\cdot\bar{6}$	$p\bar{3}$	$r\bar{3}$	$3P\bar{1}$
	46	$\not{3}12$	48	$P3(21)$	63	$(a)\cdot 3:2$	$p32$	$r32$	$3P2$
	47	$\not{3}_112$	49	$P3_1(21)$	64	$(a)\cdot 3_1:2$	$p3_12$	$r3_12$	3_1P2
	48	$\not{3}_212$	50	$P3_2(21)$	65	$(a)\cdot 3_2:2$	$p3_22$	$r3_22$	3_2P2
	49	$\not{3}m1$	46	$P3(m1)$	38	$(a)\cdot 3\cdot m$	$p3m$	$r3m$	$3mP1$
	50	$\not{3}c1$	47	$P3(c1)$	39	$(a)\cdot 3\cdot\bar{a}$	$p3a$	$r3c$	$3gP1$
	51	$\not{3}\bar{3}1m$	51	$P\bar{3}(m1)$	59	$(a)\cdot\bar{6}\cdot m$	$p\bar{3}m$	$r\bar{3}2/m$	$3mP\bar{1}2$
	52	$\not{3}\bar{3}1c$	52	$P\bar{3}(c1)$	60	$(a)\cdot\bar{6}\cdot\bar{a}$	$p\bar{3}a$	$r\bar{3}2/c$	$3gP\bar{1}2$
Hexagonal	53	$\not{6}$	56	$P6(11)$	6	$(a)\cdot 6$	$p6$	$r6$	$6P1$
	54	$\not{6}_1$	57	$P6_1(11)$	14	$(a)\cdot 6_1$	$p6_1$	$r6_1$	6_1P1
	55	$\not{6}_2$	59	$P6_2(11)$	15	$(a)\cdot 6_2$	$p6_2$	$r6_2$	6_2P1
	56	$\not{6}_3$	61	$P6_3(11)$	16	$(a)\cdot 6_3$	$p6_3$	$r6_3$	6_3P1
	57	$\not{6}_4$	60	$P6_4(11)$	17	$(a)\cdot 6_4$	$p6_4$	$r6_4$	6_4P1
	58	$\not{6}_5$	58	$P6_5(11)$	18	$(a)\cdot 6_5$	$p6_5$	$r6_5$	6_5P1
	59	$\not{6}\bar{6}$	53	$P\bar{6}(11)$	21	$(a)\cdot 3:m$	$p\bar{6}$	$r\bar{6}$	$3Pm$
	60	$\not{6}/m$	62	$P6/m(11)$	31	$(a)\cdot 6:m$	$p6/m$	$r6/m$	$6Pm$
	61	$\not{6}_3/m$	63	$P6_3/m(11)$	32	$(a)\cdot 6_3:m$	$p6_3/m$	$r6_3/m$	6_3Pm
	62	$\not{6}22$	67	$P6(22)$	70	$(a)\cdot 6:2$	$p622$	$r622$	$6P22$
	63	$\not{6}_122$	68	$P6_1(22)$	71	$(a)\cdot 6_1:2$	$p6_122$	$r6_122$	6_1P22
	64	$\not{6}_222$	70	$P6_2(22)$	72	$(a)\cdot 6_2:2$	$p6_222$	$r6_222$	6_2P22

Table 1.2.17.2 (*cont.*)

	1	2	3	4	5	6	7	8	9
	65	$\not 6_3 22$	72	$P6_3(22)$	73	$(a)\cdot 6_3:2$	$p6_322$	$r6_322$	6_3P22
	66	$\not 6_4 22$	71	$P6_4(22)$	74	$(a)\cdot 6_4:2$	$p6_422$	$r6_422$	6_4P22
	67	$\not 6_5 22$	69	$P6_5(22)$	75	$(a)\cdot 6_5:2$	$p6_522$	$r6_522$	6_5P22
	68	$\not 6mm$	64	$P6(mm)$	43	$(a)\cdot 6\cdot m$	$p6mm$	$r6mm$	$6mmP1$
	69	$\not 6cc$	65	$P6(cc)$	44	$(a)\cdot 6\cdot \bar{a}$	$p6aa$	$r6cc$	$6ggP1$
	70	$\not 6_3 mc$	66	$P6_3(cm)$	45	$(a)\cdot 6_3\cdot m$	$p6_3ma$	$r6_3mc$	6_3mgP1
	71	$\not 6\bar{6}m2$	54	$P\bar{6}(m2)$	51	$(a)\cdot m\cdot 3:m$	$p\bar{6}m2$	$r\bar{6}m2$	$3mPm2$
	72	$\not 6\bar{6}c2$	55	$P\bar{6}(c2)$	52	$(a)\cdot \bar{a}\cdot 3:m$	$p\bar{6}a2$	$r\bar{6}c2$	$3gPm2$
	73	$\not 6/mmm$	73	$P6/m(2/m2/m)$	56	$(a)\cdot m\cdot 6:m$	$p6/mmm$	$r6/m2/m2/m$	$6mmPm$
	74	$\not 6/mcc$	74	$P6/m(2/c2/c)$	57	$(a)\cdot \bar{a}\cdot 6:m$	$p6/maa$	$r6/m2/c2/c$	$6ggPm$
	75	$\not 6_3/mmc$	75	$P6_3/m(2/c2/m)$	58	$(a)\cdot m\cdot 6_3:m$	$p6_3/mma$	$r6_3/m2/m2/c$	6_3mgPm

Column 8: symbols listed by Opechowski (1986).

Column 9: symbols listed by Niggli (Chapuis, 1966).

Sets of symbols which are of a non-Hermann–Mauguin (international) type are the set of symbols in column 6 and the Niggli-type set of symbols in column 9. The set of symbols in column 8 does not use the lower-case script letter $\not$, as does *IT* A (2005), to denote a one-dimensional lattice. The order of the characters indicating symmetry elements in the set of symbols in column 7 does not follow the sequence of symmetry directions used for three-dimensional space groups. The set of symbols in column 4 have the characters indicating symmetry elements along non-lattice directions enclosed in parentheses, and do not use a lower-case script letter to denote the one-dimensional lattice. Lastly, the set of symbols in column 4, without the parentheses and with the one-dimensional lattice denoted by a lower-case script $\not$, are identical with the symbols in Part 3, or in some cases are the second setting of rod groups whose symbols are given in Part 3. These second-setting symbols are included in the symmetry diagrams of the rod groups.

1.2.17.3. Layer groups

A list of sets of symbols for the layer groups is given in Table 1.2.17.3. The information provided in the columns of this table is as follows:

Columns 1 and 2: sequential numbering and symbols used in Part 4.

Columns 3 and 4: sequential numbering and symbols listed by Wood (1964*a*,*b*) and Litvin & Wike (1991).

Columns 5 and 6: sequential numbering and symbols listed by Bohm & Dornberger-Schiff (1966, 1967).

Columns 7 and 8: sequential numbering and symbols listed by Shubnikov & Koptsik (1974) and Vainshtein (1981).

Column 9: symbols listed by Holser (1958).

Column 10: sequential numbering listed by Weber (1929).

Column 11: symbols listed by Hermann (1929*a*,*b*).

Column 12: symbols listed by Alexander & Herrmann (1929*a*,*b*).

Column 13: symbols listed by Niggli (Wood, 1964*a*,*b*).

Column 14: symbols listed by Shubnikov & Koptsik (1974).

Columns 15 and 16: symbols listed by Aroyo & Wondratschek (1987).

Column 17: symbols listed by Belov *et al.* (1957*a*,*b*).

Columns 18 and 19: symbols and sequential numbering listed by Belov & Tarkhova (1956*a*,*b*,*c*,*d*).

Columns 20 and 21: symbols listed by Cochran as listed, respectively, by Cochran (1952) and Belov & Tarkhova (1956*a*,*b*,*c*,*d*).

Column 22: symbols listed by Opechowski (1986).

Column 23: symbols listed by Grunbaum & Shephard (1987).

Column 24: symbols listed by Woods (1935*a*,*b*,*c*, 1936).

Column 25: symbols listed by Coxeter (1986).

There is also a notation for layer groups, introduced by Janovec (1981), in which all elements in the group symbol which change the direction of the normal to the plane containing the translations are underlined, *e.g.* $p4/\underline{m}$. However, we know of no listing of all layer-group types in this notation.

Sets of symbols which are of a non-Hermann–Mauguin (international) type are the sets of symbols of the Schoenflies type (columns 11 and 12) and symbols of the 'black and white' symmetry type (columns 16, 17, 18, 20, 21, 22, 24 and 25). Additional non-Hermann–Mauguin (international) type sets of symbols are those in columns 14 and 23.

Sets of symbols which do not begin with a letter indicating the lattice centring type are the sets of symbols of the Niggli type (columns 13 and 15). The order of the characters indicating symmetry elements in the sets of symbols in columns 4 and 9 does not follow the sequence of symmetry directions used for three-dimensional space groups. The set of symbols in column 6 uses parentheses to denote a symmetry direction which is not a lattice direction. In addition, the set of symbols in column 6 uses upper-case letters to denote the two-dimensional lattice of the layer group, where as in *IT* A (2005) upper-case letters denote three-dimensional lattices.

The symbols in column 8 are either identical with or, in some monoclinic and orthorhombic cases, are the second-setting or alternative-cell-choice symbols of the layer groups whose symbols are given in Part 4. These second-setting and alternative-cell-choice symbols are included in the symmetry diagrams of the layer groups.

The isomorphism between layer groups and two-dimensional magnetic space groups can be seen in Table 1.2.17.3. The set of symbols which we use for layer groups is given in column 2. The sets of symbols in columns 16, 17 and 22 are sets of symbols for the two-dimensional magnetic space groups. The basic relationship between these two sets of groups is the interexchanging of the magnetic symmetry element $1'$ and the layer symmetry element m_z. A detailed discussion of the relationship between these two sets of groups has been given by Opechowski (1986).

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