16. DIRECT METHODS

(*a*)

Figure 16.1.10.3
Success rates for the 317-atom $P2_12_12_1$ structure of gramicidin A.
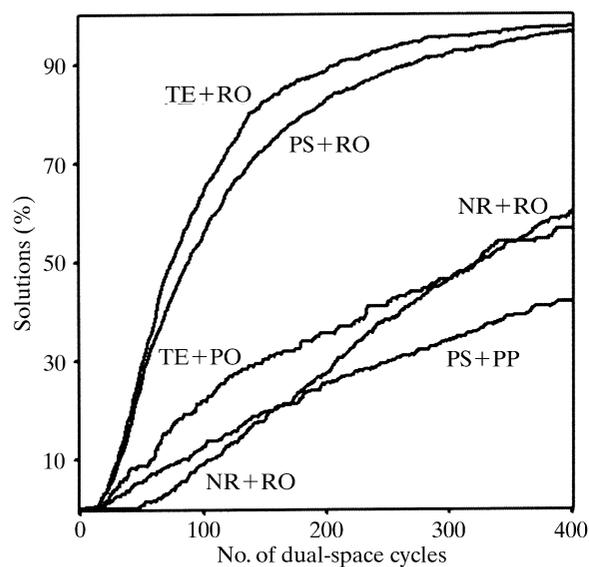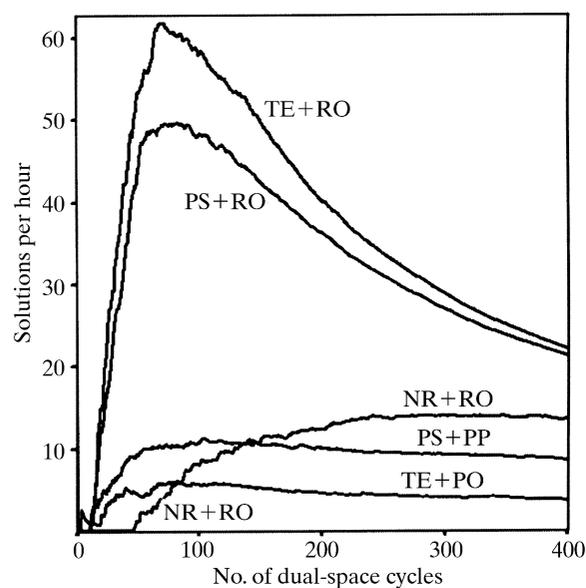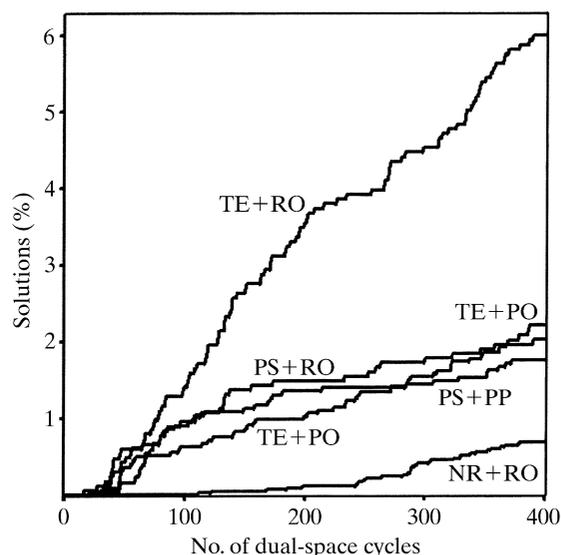

(*b*)

Figure 16.1.10.2
(*a*) Success rates and (*b*) cost effectiveness for several dual-space strategies as applied to a 148-atom $P1$ structure. The *phase-refinement strategies* are: (PS) parameter-shift reduction of the minimal-function value, (TE) Karle-type tangent expansion (holding the top 40% highest E_c fixed) and (NR) no phase refinement but Sim (1959) weights applied in the E map (these depend on E_c and so cannot be employed after phase refinement). The *real-space strategies* are: (PP) simple peak picking using $0.8N_u$ peaks, (PO) peaklist optimization (reducing N_u peaks to $2N_u/3$), and (RO) random omit maps (also reducing N_u peaks to $2N_u/3$). A total of about 10 000 trials of 400 internal loop cycles each were used to construct this diagram.

expansion appears to be even more effective (Fig. 16.1.10.3) for gramicidin A, a $P2_12_12_1$ structure (Langs, 1988). It should be noted that conventional direct methods incorporating the tangent formula tend to perform better for this space group than in $P1$, perhaps because there is less risk of a uranium-atom pseudo-solution.

Subsequent tests using *SHELXD* on several other structures have shown that the use of random omit maps is much more effective than picking the same final number of peaks from the top of the peak list. However, it should be stressed that it is the combination TE + RO that is particularly effective. A possible special case is when a very small number of atoms is sought (*e.g.* Se atoms from MAD data). Preliminary tests indicate that

peaklist optimization (PO) is competitive in such cases because the CPU time penalty associated with it is much smaller than when many atoms are involved.

With hindsight, it is possible to understand why the random omit maps provide such an efficient *search algorithm*. In macro-molecular structure refinement, it is standard practice to omit parts of the model that do not fit the current electron density well, to perform some refinement or simulated annealing (Hodel *et al.*, 1992) on the rest of the model to reduce memory effects, and then to calculate a new weighted electron-density map (omit map). If the original features reappear in the new density, they were probably correct; in other cases the omit map may enable a new and better interpretation. Thus, random omit maps should not lead to the loss of an essentially correct solution, but enable efficient searching in other cases. It is also interesting to note that the results presented in Figs. 16.1.10.2 and 16.1.10.3 show that it is possible, albeit much less efficiently, to solve both structures using random omit maps without the use of any phase relationships based on probability theory (curves NR + RO).

16.1.10.3. Expansion to P1

The results shown in Table 16.1.1.1 and Fig. 16.1.10.2 indicate that success rates in space group $P1$ can be anomalously high. This suggests that it might be advantageous to expand all struc-tures to $P1$ and then to locate the symmetry elements afterwards. However, this is more computationally expensive than per-forming the whole procedure in the true space group, and in practice such a strategy is only competitive in low-symmetry space groups such as $P2_1$, $C2$ or $P\bar{1}$ (Chang *et al.*, 1997). Expan-sion to $P1$ also offers some opportunities for starting from 'slightly better than random' phases. One possibility, successfully demonstrated by Sheldrick & Gould (1995), is to use a rotation search for a small fragment (*e.g.* a short piece of α-helix) to generate many sets of starting phases; after expansion to $P1$ the translational search usually required for molecular replacement is not needed. Various Patterson superposition minimum func-tions (Sheldrick & Gould, 1995; Pavelčík, 1994) can also provide an excellent start for phase determination for data expanded to $P1$. Drendel *et al.* (1995) were successful in solving small organic structures *ab initio* by a Fourier recycling method using perturbed Fourier amplitudes and data expanded to $P1$ without the use of

references