speed v is

$$E_e = \frac{m_e c^2}{(1 - v^2/c^2)^{1/2}} = \gamma m_e c^2,$$

where m_e is the rest mass of the electron, 9.10938356 (11) $\times$ 10^{-31} kg. The term $1/(1 - v^2/c^2)^{1/2}$ is referred to as γ and is the factor by which the mass of the electron increases from its rest mass because of its relativistic speed. Expressed in eV (the conversion factor from kg to eV is c^2/e), the electron rest mass is 5.109989461 (31) $\times$ 10^5 eV, so that

$$\gamma \simeq 1957 E_e \text{ [GeV]}$$

when E_e is given in the customary units of GeV. Thus for a 3-GeV machine, a common energy for a synchrotron-radiation source, γ has the value of 5871. The mass of an electron with energy 3 GeV is therefore 3.22 atomic mass units, so around 7% more massive than a stationary atom of ^{3}H or ^{3}He.

Electrons do not circulate individually in the storage ring but in a series of bunches that are in phase with the accelerating radio frequency. Radiation is therefore emitted in pulses as each bunch passes through a bending magnet or insertion device. Thus the number and distribution of the electron bunches around the orbit determine the time structure of the emitted radiation. For most powder-diffraction applications using synchrotron radiation, the pulsed nature of the source can be neglected and the radiation can be regarded as continuous, although attention should also be paid to the performance of detectors that are more susceptible to pulse pile-up problems when the radiation arrives at very high average rates or in concentrated bursts (Cousins, 1994; Laundy & Collins, 2003; Honkimäki & Suortti, 2007), which can happen with certain bunch-filling modes. Certain specialized experiments requiring very fast time resolution can exploit the time structure of the source. In such experiments the longitudinal dimension of the bunches controls the pulse duration, which is usually a few tens of picoseconds.

In discussing the performance of different beamlines, the spectral brightness (Mills *et al.*, 2005) is often quoted for the source and is defined as

spectral brightness =
photons per second per 0.1% bandwidth per mrad^2 per mm^2,

where '0.1% bandwidth' represents $\delta\lambda/\lambda = 0.001$, the mrad^2 term expresses the solid-angle of the emission of the X-rays from the source and the mm^2 term relates to the cross-sectional area of the source. Thus a source of high spectral brightness emits many photons per second of the specified energy, into a narrow solid angle, with a small source size. The source size, which may well differ in the horizontal and vertical directions, is an important consideration as source size and beam divergence ultimately limit the performance of the beamline optical system in terms of collimation, energy resolution and focal spot size.

2.2.2.1. *Bending magnets*

A bending magnet provides a vertical magnetic field to deflect the electrons laterally in the horizontal plane from a straight-line trajectory, and thereby causes the emission of synchrotron radiation (see Fig. 2.2.2). The lateral Lorentz force, F, acting on an electron travelling at velocity v in a magnetic field B is mutually perpendicular to both the magnetic field and the direction of travel of the electron, and is given by

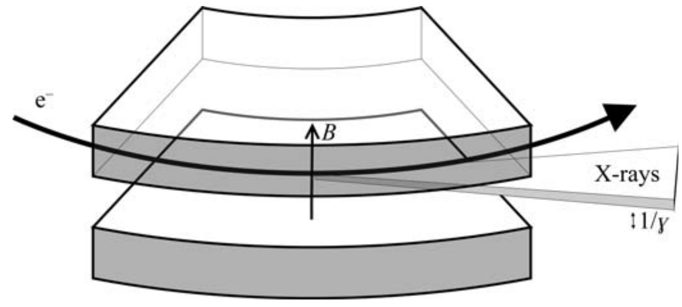

Figure 2.2.2
Emission of a fan of radiation by the electron beam as it curves in a bending magnet from one straight section of the ring to the next.

$$F = evB.$$

In a bending magnet the magnetic field is applied over an extended distance leading to a curved path of radius ρ. The centripetal acceleration is $F/\gamma m_e$, which for circular motion is equal to v^2/ρ. Since $v \simeq c$,

$$\rho = \frac{\gamma m_e c}{eB},$$

so the radius of curvature decreases with magnetic field strength and increases with machine energy *via* increased γ. With the electron energy expressed in GeV, this can be approximated to $\rho \simeq 3.34 E_e$ [GeV]$/B$ (where $10^9/c \simeq 3.34$).

Synchrotron radiation is emitted in a forward cone tangential to the direction of the electrons' motion (Fig. 2.2.3) with a nominal Gaussian distribution and an opening angle of the order of $1/\gamma$. Thus the radiation is highly collimated in the vertical plane. In the horizontal plane, synchrotron radiation is emitted in a broad fan, tangential to the curved trajectory of the electrons as they sweep through the bending magnet. Only a fraction of the radiation emitted by a bending magnet enters the associated beamline *via* a cooled aperture defining a horizontal acceptance angle of a few mrad. The radiation is polarized in the plane of the synchrotron orbit. Sometimes, more than one beamline can be built on a bending magnet with a suitable angular separation between them.

Photons are emitted over a broad spectral range. The critical photon energy, ε_c, divides the emitted power into equal halves and is given by

$$\varepsilon_c = \frac{3\hbar c \gamma^3}{2\rho} = \frac{3\hbar \gamma^2 eB}{2m_e} = \frac{3\hbar e E_e^2 B}{2m_e^3 c^4} = 4.151 E_e^2 B,$$

or, with photon and electron energies in keV and GeV, respectively,

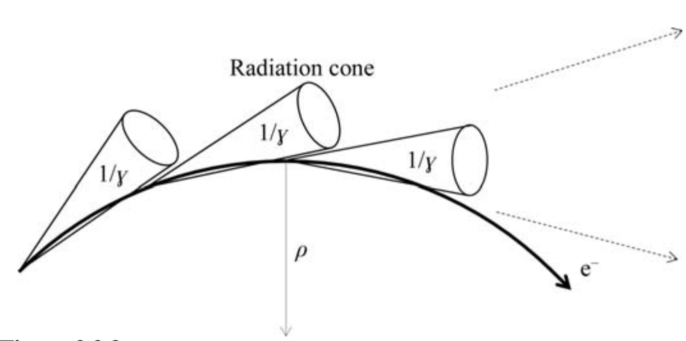

Figure 2.2.3
Synchrotron radiation is emitted in a cone of opening angle of the order of $1/\gamma$ tangential to the electrons as they follow a curved trajectory through the bending magnet.