International Tables for Crystallography (2018). Vol. H, Section 2.2.2.2.1, p. 53.

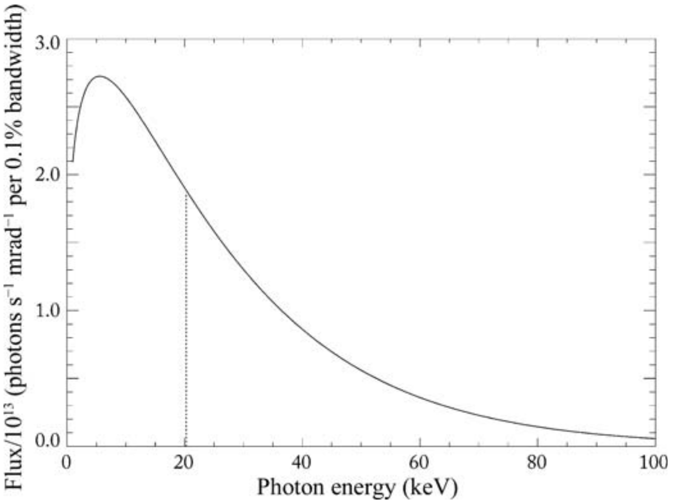

Figure 2.2.4
Spectrum of a bending magnet ($B = 0.85$ T) at the ESRF with an electron energy of 6 GeV ($\gamma = 11\,742$), shown as flux per horizontal mrad for a 0.1% energy bandwidth at a storage-ring current of 200 mA. The critical energy of 20.3 keV divides the emitted power into equal halves.

$$\varepsilon\ [\text{keV}] = 0.665E_e^2\ [\text{GeV}]B.$$

The higher the critical energy, the greater the number of photons produced with short X-ray wavelengths. As an example, consider a bending magnet at the ESRF in Grenoble, France, which has a 6-GeV storage ring and bending magnets with a field of 0.85 T. The bending radius is 23.5 m and the critical photon energy is 20.3 keV (equivalent to a wavelength of 0.61 Å). The spectrum of such a device is shown in Fig. 2.2.4.

The vertical collimation of the radiation varies with photon energy in a nonlinear manner (Kim, 2001). Nevertheless, the divergence decreases with increased photon energy, so beams with the shortest wavelengths are the most vertically collimated. Various approximations can be written to describe the variation, such as for a single electron (Margaritondo, 1988),

$$\sigma_v(\varepsilon) \simeq \frac{0.565}{\gamma}\left(\frac{\varepsilon_c}{\varepsilon}\right)^{0.425},$$

where $\sigma_v(\varepsilon)$ is the standard deviation of the vertical-divergence distribution of photons of energy ε. For a population of electrons circulating in a storage ring, the distribution of the trajectories with respect to the plane of the orbit (of the order µrad) must also be considered, as these add to the vertical emission distribution. An approximation such as

$$\Psi_v(\varepsilon) = 2\sigma_v(\varepsilon) \simeq \frac{1.2}{\gamma}\left(\frac{\varepsilon_c}{\varepsilon}\right)^{1/2}$$

will often be adequate to estimate the vertical divergence Ψ_v in the vicinity of ε_c. Thus for the bending magnet illustrated in Fig. 2.2.4, photons at the critical energy of 20.3 keV will have a vertical divergence of ∼100 µrad. A beamline would probably accept less than this, *e.g.* a 1.5-mm-high slit at 25 m from the source defining the beam onto a monochromator crystal defines an angle of ∼60 µrad.

2.2.2.2. *Insertion devices*

Insertion devices can be classified into two main types, termed 'wigglers' and 'undulators', illustrated in Fig. 2.2.5. A wiggler has a relatively long magnetic period and the radiation from each oscillation is emitted like a series of powerful bending magnets, summing together to provide increased intensity. An undulator has a relatively short magnetic period and the radiation from sequential oscillations interferes coherently to give modified beam characteristics.

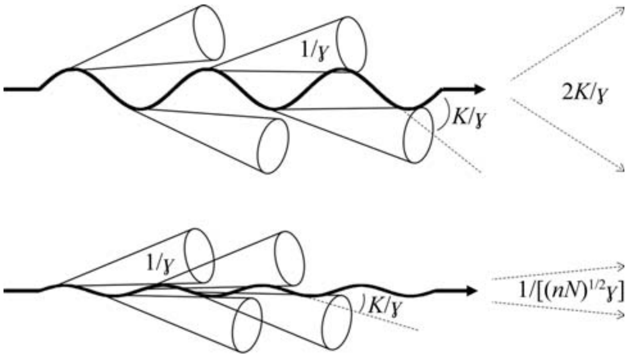

Figure 2.2.5
Schematic illustration of a wiggler (upper) and an undulator (lower).

For insertion devices the magnetic field acting on the electrons varies sinusoidally along the device,

$$B(z) = B_0 \sin(2\pi z/\lambda_u),$$

where B_0 is the peak magnetic field, z is the distance along the insertion-device axis and λ_u is the magnetic period. With a vertical field, the alternating magnetic field causes the electron path to oscillate in the horizontal plane. Note that the radiation is emitted mainly towards the outsides of the oscillations where the electrons change transverse direction, and where the magnetic field and beam-path curvature are highest. The maximum angular deflection of an electron from the axis of the insertion device is K/γ, where the deflection parameter K is given by

$$K = \frac{eB_0\lambda_u}{2\pi m_e c},$$

which simplifies to $K = 0.0934B_0\lambda_u$ [mm] with λ_u expressed in mm. K is a crucial parameter that determines the behaviour of the insertion device.

2.2.2.2.1. *Wigglers*

If K is large (10 or above), the insertion device is a wiggler and the electrons oscillate with an amplitude significantly greater than the emitted radiation's natural opening angle $1/\gamma$. Every oscillation along the device produces a burst of synchrotron radiation and these add together incoherently so increasing the flux in proportion to the number of magnetic periods. The radiation emerges from the wiggler in a horizontal fan with a horizontal opening angle $\sim 2K/\gamma$. The intensity of a wiggler-based beamline can be very high because each oscillation produces synchrotron radiation, and this radiation is directed close to the axis of the device. Like a bending magnet, wigglers produce a continuous spectrum but with the critical energy shifted to harder energies because the magnetic field is (usually) greater. Thus for a wiggler at a 6-GeV source, with a magnetic field of 1.2 T and a magnetic period of 125 mm, K is 14, the maximum deflection of the electrons from the straight-line path is 1.2 mrad and the critical photon energy is 28.7 keV. Magnetic fields of several tesla can be exploited using superconducting magnets to obtain even higher critical photon energies.

2.2.2.2.2. *Undulators*

If the value of K is 2 or less, the insertion device is an undulator. The deflection of the electrons is comparable to the natural opening angle of the emitted radiation $1/\gamma$. Radiation emitted from sequential oscillations interferes coherently, and the beam