2.5. TWO-DIMENSIONAL POWDER DIFFRACTION

where $F(x, y)$ is the flux (in photons s^{-1}) intercepted by the pixel and B is the brightness of the source (in photons s^{-1} mrad^{-2}) or scattering from the sample. The ratio of the flux in pixel $P(x, y)$ to that in the centre pixel $P(0, 0)$ is then given as

$$\frac{F(x, y)}{F(0, 0)} = \frac{D^3}{R^3} = \frac{D^3}{(D^2 + x^2 + y^2)^{3/2}} = \cos^3\phi, \quad (2.5.20)$$

where ϕ is the angle between the X-rays to the pixel $P(x, y)$ and the line from S to the detector in perpendicular direction. It can be seen that the greater the sample-to-detector distance, the smaller the difference between the centre pixel and the edge pixel in terms of the flux from the homogeneous source. This is the main reason why a data frame collected at a short sample-to-detector distance has a higher contrast between the edge and centre than one collected at a long sample-to-detector distance.

2.5.3.2.2. Spatial resolution of area detectors

In a 2D diffraction frame, each pixel contains the X-ray intensity collected by the detector corresponding to the pixel element. The pixel size of a 2D detector can be determined by or related to the actual feature sizes of the detector structure, or artificially determined by the readout electronics or data-acquisition software. Many detector techniques allow multiple settings for variable pixel size, for instance a frame of 2048×2048 pixels or 512×512 pixels. Then the pixel size in 512 mode is 16 (4×4) times that of a pixel in 2048 mode. The pixel size of a 2D detector determines the space between two adjacent pixels and also the minimum angular steps in the diffraction data, therefore the pixel size is also referred to as pixel resolution.

The pixel size does not necessarily represent the true spatial resolution or the angular resolution of the detector. The resolving power of a 2D detector is also limited by its point-spread function (PSF) (Bourgeois *et al.*, 1994). The PSF is the two-dimensional response of a 2D detector to a parallel point beam smaller than one pixel. When the sharp parallel point beam strikes the detector, not only does the pixel directly hit by the beam record counts, but the surrounding pixels may also record some counts. The phenomenon is observed as if the point beam has spread over a certain region adjacent to the pixel. In other words, the PSF gives a mapping of the probability density that an X-ray photon is recorded by a pixel in the vicinity of the point where the X-ray beam hits the detector. Therefore, the PSF is also referred to as the spatial redistribution function. Fig. 2.5.12(*a*) shows the PSF produced from a parallel point beam. A plane at half the maximum intensity defines a cross-sectional region within the PSF. The FWHM can be measured at any direction crossing the centroid of the cross section. Generally, the PSF is isotropic, so the FWHMs measured in any direction should be the same.

Measuring the PSF directly by using a small parallel point beam is difficult because the small PSF spot covers a few pixels and it is hard to establish the distribution profile. Instead, the line-spread function (LSF) can be measured with a sharp line beam from a narrow slit (Ponchut, 2006). Fig. 2.5.12(*b*) is the intensity profile of the image from a sharp line beam. The LSF can be obtained by integrating the image from the line beam along the direction of the line. The FWHM of the integrated profile can be used to describe the LSF. Theoretically, LSF and PSF profiles are not equivalent, but in practice they are not distinguished and may be referenced by the detector specification interchangeably. For accurate LSF measurement, the line beam is intentionally positioned with a tilt angle from the orthogonal
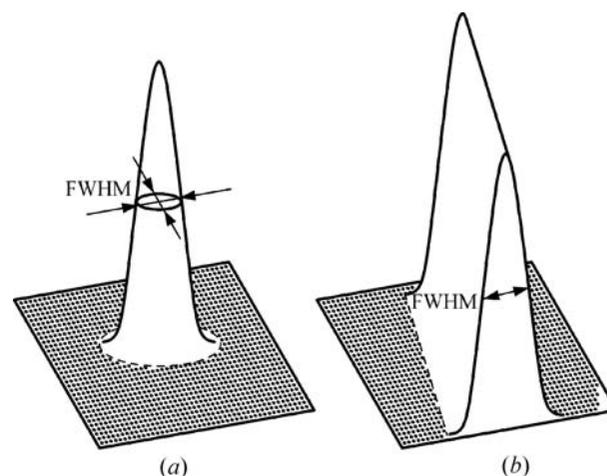

Figure 2.5.12
(*a*) Point-spread function (PSF) from a parallel point beam; (*b*) line-spread function (LSF) from a sharp line beam.

direction of the pixel array so that the LSF can have smaller steps in the integrated profile (Fujita *et al.*, 1992).

The RMS (root-mean-square) of the distribution of counts is another parameter often used to describe the PSF. The normal distribution, also called the Gaussian distribution, is the most common shape of a PSF. The RMS of a Gaussian distribution is its standard deviation, σ. Therefore, the FWHM and RMS have the following relation, assuming that the PSF has a Gaussian distribution:

$$\text{FWHM} = 2[-2\ln(1/2)]^{1/2}\text{RMS} = 2.3548 \times \text{RMS}. \quad (2.5.21)$$

The values of the FWHM and RMS are significantly different, so it is important to be precise about which parameter is used when the value is given for a PSF.

For most area detectors, the pixel size is smaller than the FWHM of the PSF. The pixel size should be small enough that at least a 50% drop in counts from the centre of the PSF can be observed by the pixel adjacent to the centre pixel. In practice, an FWHM of 3 to 6 times the pixel size is a reasonable choice if use of a smaller pixel does not have other detrimental effects. A further reduction in pixel size does not necessarily improve the resolution. Some 2D detectors, such as pixel-array detectors, can achieve a single-pixel PSF. In this case, the spatial resolution is determined by the pixel size.

2.5.3.2.3. Detective quantum efficiency and energy range

The detective quantum efficiency (DQE), also referred to as the detector quantum efficiency or quantum counting efficiency, is measured by the percentage of incident photons that are converted by the detector into electrons that constitute a measurable signal. For an ideal detector, in which every X-ray photon is converted to a detectable signal without additional noise added, the DQE is 100%. The DQE of a real detector is less than 100% because not every incident X-ray photon is detected, and because there is always some detector noise. The DQE is a parameter defined as the square of the ratio of the output and input signal-to-noise ratios (SNRs) (Stanton *et al.*, 1992):

$$\text{DQE} = \left(\frac{(S/N)_{\text{out}}}{(S/N)_{\text{in}}}\right)^2. \quad (2.5.22)$$

The DQE of a detector is affected by many variables, for example the X-ray photon energy and the counting rate. The dependence of the DQE on the X-ray photon energy defines the