

## 2. INSTRUMENTATION AND SAMPLE PREPARATION

The plane given by  $X_L$  and  $Y_L$  is the diffractometer plane. The axis  $Z_L$  is perpendicular to the diffractometer plane. The axes  $X_L$ ,  $Y_L$  and  $Z_L$  form a right-handed rectangular coordinate system with the origin at the instrument centre. The incident X-ray beam propagates along the  $X_L$  axis, which is also the rotation axis of all diffraction cones. The apex angles of the cones are determined by the  $2\theta$  values given by the Bragg equation. The apex angles are twice the  $2\theta$  values for forward reflection ( $2\theta \leq 90^\circ$ ) and twice the value of  $180^\circ - 2\theta$  for backward reflection ( $2\theta > 90^\circ$ ). For clarity, only one diffraction cone of forward reflection is displayed. The  $\gamma$  angle is the azimuthal angle from the origin at the six o'clock direction with a right-handed rotation axis along the opposite direction of incident beam ( $-X_L$  direction). A given  $\gamma$  value defines a half plane with the  $X_L$  axis as the edge; this will be referred to as the  $\gamma$  plane hereafter. The diffractometer plane consists of two  $\gamma$  planes at  $\gamma = 90^\circ$  and  $\gamma = 270^\circ$ . Therefore many equations developed for 2D-XRD should also apply to conventional XRD if the  $\gamma$  angle is given as a constant of  $90^\circ$  or  $270^\circ$ . A pair of  $\gamma$  and  $2\theta$  values represents the direction of a diffracted beam. The  $\gamma$  angle takes a value of 0 to  $360^\circ$  for a complete diffraction ring with a constant  $2\theta$  value. The  $\gamma$  and  $2\theta$  angles form a spherical coordinate system which covers all the directions from the origin of sample (instrument centre). The  $\gamma$ - $2\theta$  system is fixed in the laboratory system  $\mathbf{X}_L$ ,  $\mathbf{Y}_L$ ,  $\mathbf{Z}_L$ , which is independent of the sample orientation and detector position in the goniometer.  $2\theta$  and  $\gamma$  are referred to as the diffraction-space parameters. In the laboratory coordinate system  $\mathbf{X}_L$ ,  $\mathbf{Y}_L$ ,  $\mathbf{Z}_L$ , the surface of a diffraction cone can be mathematically expressed as

$$y_L^2 + z_L^2 = x_L^2 \tan^2 2\theta, \quad (2.5.1)$$

with  $x_L \geq 0$  or  $2\theta \leq 90^\circ$  for forward-diffraction cones and  $x_L < 0$  or  $2\theta > 90^\circ$  for backward-diffraction cones.

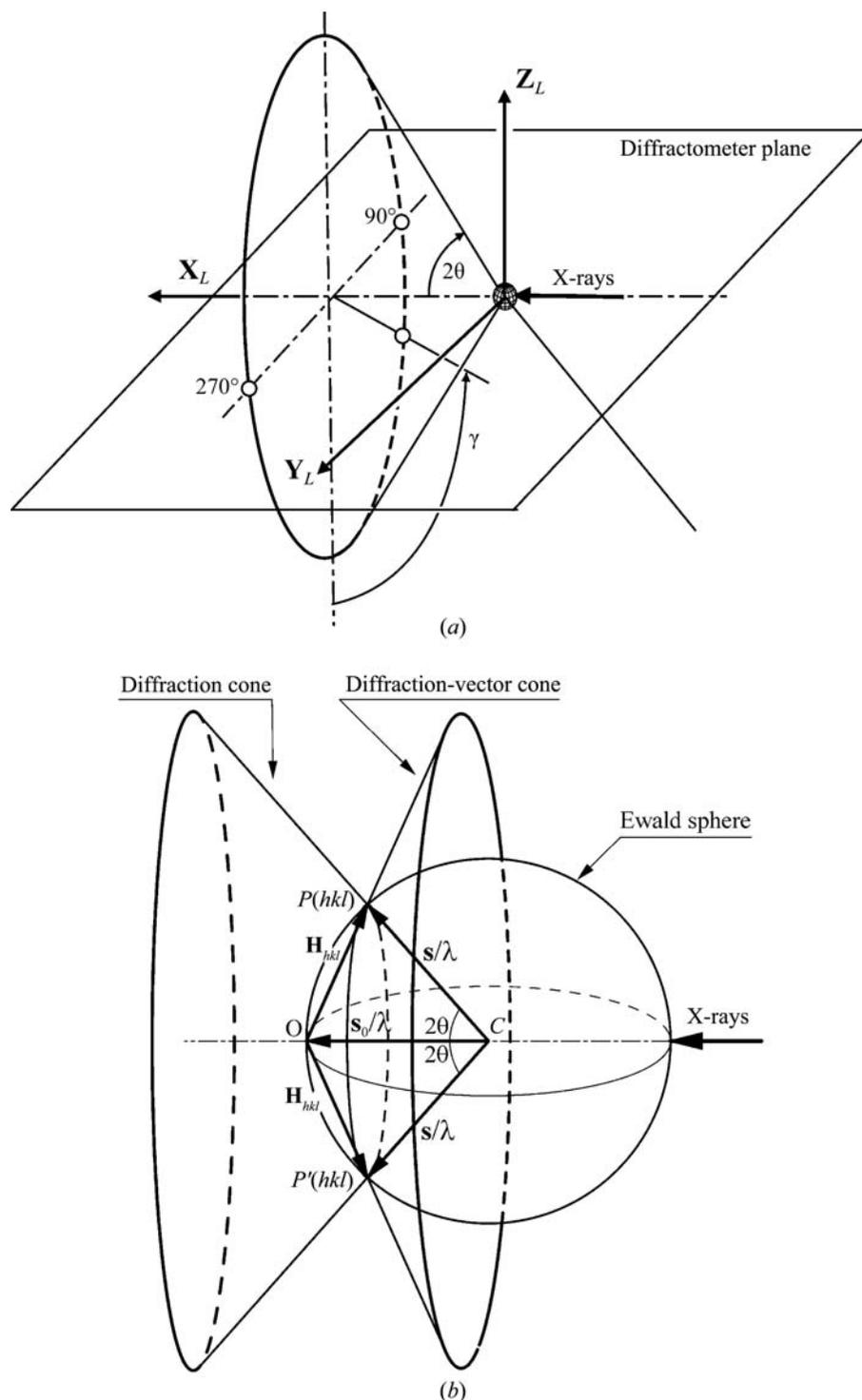
#### 2.5.2.1.2. Diffraction-vector cones in laboratory coordinates

Fig. 2.5.3(b) shows the diffraction-vector cone corresponding to the diffraction cone in the laboratory coordinate system.  $C$  is the centre of the Ewald sphere. The diffraction condition can be given by the Laue equation as

$$\frac{\mathbf{s} - \mathbf{s}_0}{\lambda} = \mathbf{H}_{hkl}, \quad (2.5.2)$$

where  $\mathbf{s}_0$  is the unit vector representing the incident beam,  $\mathbf{s}$  is the unit vector representing the diffracted beam and  $\mathbf{H}_{hkl}$  is the reciprocal-lattice vector. Its magnitude is given as

$$\left| \frac{\mathbf{s} - \mathbf{s}_0}{\lambda} \right| = \frac{2 \sin \theta}{\lambda} = |\mathbf{H}_{hkl}| = \frac{1}{d_{hkl}}, \quad (2.5.3)$$



**Figure 2.5.3**  
 The diffraction cone and the corresponding diffraction-vector cone.

in which  $d_{hkl}$  is the  $d$ -spacing of the crystal planes ( $hkl$ ). It can be easily seen that it is the Bragg law in a different form. Therefore, equation (2.5.2) is the Bragg law in vector form. In the Bragg condition, the vectors  $\mathbf{s}_0/\lambda$  and  $\mathbf{s}/\lambda$  make angles  $\theta$  with the diffracting planes ( $hkl$ ) and  $\mathbf{H}_{hkl}$  is normal to the ( $hkl$ ) crystal plane. In order to analyse all the X-rays measured by a 2D detector, we extend the concept to all scattered X-rays from a sample regardless of the Bragg condition. Therefore, the index ( $hkl$ ) can be removed from the above expression.  $\mathbf{H}$  is then a vector which takes the direction bisecting the incident beam and the scattered beam, and has dimensions of inverse length given by  $2 \sin \theta/\lambda$ . Here  $2\theta$  is the scattering angle from the incident beam. The vector  $\mathbf{H}$  is referred to as the scattering vector or, alternatively, the diffraction vector. When the Bragg condition is