3. METHODOLOGY

Table 3.4.3
Classification of indexing methods

Method (strategy/search)	Space	Exhaustive
Zone indexing	Parameter	No
SIW heuristic	Parameter	Semi
Scan/covariance	Parameter (for cubic to monoclinic); index (for triclinic)	To monoclinic
Index heuristics	Index	Semi
Index permutation	Index	Yes
Successive dichotomy	Parameter	Yes
Grid search	Parameter	Yes
Genetic algorithms	Parameter	No
Monte Carlo	Parameter	No

made by considering whether a program works in direct or reciprocal space, or uses Bragg diffraction line positions or the whole experimental diffraction profile. Using the whole experimental diffraction profile requires a lot of computing time, but has become possible as a consequence of recent increases in the speed of computers. In the following, descriptions of the principal (default) steps of the most widely used indexing programs are given. Several non-default options are available for each program. The chances of success of the indexing step increase if more than one program is used.

3.4.4.1. *Traditional indexing programs*

3.4.4.1.1. *ITO (Visser, 1969)*

This program is based on the zone-indexing strategy and uses the Runge–Ito–de Wolff–Visser method of decomposition of the reciprocal space into zones, as described in Section 3.4.3.1.1.

The following steps are executed by the program:

(1) The potential zones are found always using the 20 lowest Bragg angle peaks. The program does not work with fewer peaks.

(2) All possible combinations of the six best zones (including the combination of each zone with itself) are found by searching for trial zones that share a row of common points. For every pair of such zones, the angle between them is found, thus giving a trial reciprocal lattice.

(3) The reduction of the resulting unit cells is carried out using the Delaunay–Ito method (Pecharsky & Zavalij, 2009).

(4) The program tries to index the first 20 lines and repeats this check after least-squares refinement of the unit-cell parameters.

(5) The figures of merit are calculated to assess the quality of each trial unit cell and the four best lattices are provided.

ITO is very efficient at indexing patterns with low symmetry and is only weakly sensitive to impurity peaks, if they occur at high angles. The most frequent causes of failure are inaccuracy or incompleteness of the input data.

3.4.4.1.2. *TREOR90 (Werner et al., 1985)*

Classified by Shirley (1980, 2003) as semi-exhaustive, *TREOR90* is based on the index-heuristics strategy (see Section 3.4.3.1.3) and uses a trial-and-error approach. It performs the following steps:

(1) Some basis lines are selected among the experimental *d* values, generally from the low-2θ region of the powder diffraction pattern. Five sets of basis lines are generally sufficient for orthorhombic tests, whereas more than seven

sets may be necessary for the monoclinic system. At least 20–25 experimental *d* values are potentially required.

(2) The trial unit cells in the index space are searched by varying the Miller indices that are tentatively assigned to the basis lines.

(3) The analysis starts with cubic symmetry and, in a stepwise manner, tests for lower-symmetry crystal systems are performed. In the case of monoclinic symmetry, a special short-axis test is carried out.

(4) The solution of the linear system in equation (3.4.8) gives the possible cell parameters. Different combinations of the basis lines are tested.

(5) Each possible solution is checked by using the full list of experimental lines.

(6) The quality of the trial cell parameters is mainly assessed by using the M_{20} figure of merit [see equation (3.4.4)]. An effective rule for identifying a reliable solution is $M_{20} > 10$ and no more than one unindexed line.

The success of the program is related to the use of some suitable standard sets of parameter values (maximum unit-cell volume, maximum cell axis, tolerance of values *etc.*) arising from the accumulated experience of the authors; they can be easily changed by the user *via* suitable keywords in the input file.

3.4.4.1.3. *DICVOL91 (Boultif & Louër, 1991)*

This program works in direct space (down to the monoclinic system) by using the successive-dichotomy search method (see Section 3.4.3.1.5), which was introduced for the automatic indexing of powder diffraction patterns by Louër & Louër (1972). *DICVOL91* has been defined as exhaustive by Shirley (1980, 2003). Its main steps are:

(1) The unit-cell volume is partitioned by moving from high to low symmetry. Shells of 400 Å^3 of volume are scanned for all the symmetry systems except for triclinic; for the triclinic system the shells are based on the definition of volume proposed by Smith (1977) (see Section 3.4.3.1.5).

(2) A search using the successive-dichotomy method, based on suitable intervals (see Section 3.4.3.1.5), is carried out. In the case of the triclinic system the general expression for $Q(hkl)$ as function of the direct cell parameters is too complicated, and the reciprocal-space parameters in equation (3.4.2) are used for setting the intervals.

(3) The derived cell parameters are refined using the least-squares method.

(4) The quality of each trial unit cell is evaluated by using the M_N and F_N figures of merit (see Section 3.4.2.1).

The program is fast at performing exhaustive searches in parameter space (except for the triclinic case); on the other hand, its efficiency is strongly related to the quality of the data and to the presence of impurities (in fact, impurities are not permitted).

3.4.4.2. *Evolved indexing programs*

3.4.4.2.1. *N-TREOR09 (Altomare et al., 2009)*

Implemented in the *EXPO* program (Altomare *et al.*, 2013) to perform the powder pattern indexing step, *N-TREOR09* is an update of *N-TREOR* (Altomare *et al.*, 2000), which in turn is an evolution of *TREOR90*, and preserves the main strategies with some changes introduced to make the program more exhaustive and powerful. In particular:

(*a*) If the default indexing process fails, the unit-cell search is automatically repeated by changing some default choices, *e.g.*, increasing the tolerance value on the observed *d* values. If still no solution is obtained, the maximum (*hkl*) Miller indices assigned to the orthorhombic or monoclinic base lines are increased and the tolerance limits of the default values are halved in order to avoid the generation of wrong large unit cells.

(*b*) At the end of the first run, whatever the obtained results, a possible 2θ zero-position shift is taken into account: the indexing process starts again by applying positive and negative 2θ zero-position shifts to the original peak search result.

(*c*) An exhaustive triclinic search is performed. The dominant-zone tests that are usually carried out for the monoclinic system have been extended to include the triclinic case.

(*d*) A new figure of merit, WRIP20, more powerful than the classical M_{20}, is used. It is calculated when more than one possible cell is found and takes into account the M_{20} value, the full experimental pattern, the degree of reflection overlap, the systematically absent reflections and the number of unindexed lines (see Section 3.4.2.1).

This program is also able to index powder patterns from small proteins: see Example 4 in Section 3.4.4.6.2.

3.4.4.2.2. *DICVOL06 (Louër & Boultif, 2006, 2007) and DICVOL14 (Louër & Boultif, 2014)*

The most recent of a series of versions, *DICVOL14* is the successor of *DICVOL04* (Boultif & Louër, 2004) and *DICVOL06*. *DICVOL06* includes *DICVOL04* with its optimized search procedure and an extended search in shells of volumes. *DICVOL04* represented an improvement of *DICVOL91*. Among the features of *DICVOL06* are:

(*a*) A tolerance for unindexed lines that can result from the presence of unwanted additional phases or inaccurately measured peaks. The program can tolerate a user-defined number of unindexed lines. Care must be taken when using this option to avoid the possibility of generating erroneous cells. It is worth noting that the inclusion of the possibility of at least one unindexed peak has markedly increased the success rate of *DICVOL06*.

(*b*) A correction of the zero-point error in the measured data. *Via* an *a priori* zero-origin evaluation, two different approaches can be followed: (i) if there is a non-negligible zero shift (*i.e.*, ~0.1°), the reflection-pair method is adopted (Dong *et al.*, 1999); (ii) if the shift is small (<0.03°), a refinement of the experimental data zero point together with the cell parameters is carried out as soon as a solution is found. In the monoclinic and triclinic systems, a reduced-cell analysis is performed to choose among equivalent solutions.

(*c*) When a solution is found in a 400 Å^3 shell of volume, the exhaustive search is extended to the whole domain.

No formal limits on the number of input Bragg peaks have been established but, for reliable indexing, it is recommended that 20 or more peaks (in the low-2θ region) are used.

Compared to *DICVOL04/DICVOL06*, *DICVOL14* includes: an optimization of filters in the final stages of the convergence of the successive dichotomy process; an optimization and extension of scanning limits for the triclinic case; a new approach for zero-point offset evaluation; a detailed review of the input data from the resulting unit cells; and cell centring tests. *DICVOL14* has been improved particularly for triclinic cases, which are generally the most difficult to solve with the dichotomy algorithm.

3.4.4.3. *Non-traditional indexing programs*

The indexing programs described above are based on using, for a limited number of lines, the measured positions of peak maxima as directly obtained from the experimental powder diffraction pattern. *Conograph* (Oishi-Tomiyasu, 2014*b*), which has been more recently proposed, also belongs to that group of programs. A brief description of *Conograph* follows

3.4.4.3.1. *Conograph: indexing via the topographs method*

Conograph is based on the topographs method, and its main functions are the determination of the primitive unit cell and lattice symmetry, and refinement of lattice parameters. Among the main features we note:

(1) A new Bravais-lattice determination algorithm (Oishi-Tomiyasu, 2012), which has been proved to be stable with respect to peak-position errors under very general conditions. The algorithm applies the Minkowski reduction to primitive cells and the Delaunay reduction (Delaunay, 1933) to face-centred, body-centred, rhombohedral and base-centred cells in such a way that the computational efficiency of the process is better than the Andrews & Bernstein (1988) method.

(2) The two figures of merit M_n^{Rev} and M_n^{Sym} proposed by Oishi-Tomiyasu (2013) are used for for selecting the true unit cells. They are also used to estimate the zero-point shift.

(3) The use of many observed peaks in the default setting, which aims to make *Conograph* robust against dominant zones and missing or false peaks (Oishi-Tomiyasu, 2014*b*).

(4) The method for exhaustively searching unit cells that involve geometrical ambiguity (Oishi-Tomiyasu, 2014*a*, 2016). The geometrical ambiguities that are detected also include lattices with very similar calculated lines, because of the error tolerance in the *d* spacings.

Programs that use only the measured positions of peak maxima are particularly vulnerable to experimental errors in the measured peak positions and to the presence of impurity peaks. For these reasons, at the end of the 1990s new indexing strategies were developed that do not require the peak locations in the experimental pattern. These approaches are completely different from the methods described above because they use the whole diffraction profile. They try to explore the parameter space (direct space) exhaustively by applying different optimization techniques in order to find the cells in best agreement with the experimental powder diffraction pattern. Some of the most widely used indexing programs in direct space are described here.

3.4.4.3.2. *GAIN: indexing via a genetic-algorithm search method*

The use of genetic algorithms (GAs) for indexing powder diffraction data by exploiting the diffraction geometry (as in the traditional indexing methods) was firstly proposed by Tam & Compton (1995) and Paszkowicz (1996). Subsequently, Kariuki *et al.* (1999) applied GA techniques by using whole profile fitting with the aim of exploring the parameter space $\{a, b, c, \alpha, \beta, \gamma\}$ and finding the global minimum of the *R*-factor $\{a, b, c, \alpha, \beta, \gamma\}$ hypersurface, yielding the parameter set able to generate the best agreement between the observed and calculated powder diffraction patterns.

This new strategy has been implemented in the program *GAIN* (Harris *et al.*, 2000), whose main features are:

(1) Starting from a population of N_p sets of lattice parameters and using the evolutionary operations of mating, mutation and natural selection, the population is allowed to evolve

references